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MULTIVARIABLE ADAPTIVE CONTROL: UNCERTAIN HIGH FREQUENCY GAIN, ROBUSTNESS AND GUARANTEED TRANSIENT PERFORMANCE

Andrei Giordano Holanda Battistel

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Orientadores: Liu Hsu
Eduardo Vieira Leão Nunes

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Examinada por:

Prof. Liu Hsu, Docteur d'Etat.

Prof. Eduardo Vieira Leão Nunes, D. Sc.

Prof. José Paulo Vilela Soares da Cunha, D. Sc.

Prof. Ramon Romankevicius Costa, D. Sc.

Prof. Alexandre Sanfelice Bazanela, D. Sc.

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CONTROLE ADAPTATIVO MULTIVARIÁVEL: GANHO DE ALTA FREQUÊNCIA INCERTO, ROBUSTEZ E DESEMPENHO TRANSITÓRIO GARANTIDO

Andrei Giordano Holanda Battistel

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Orientadores: Liu Hsu
Eduardo Vieira Leão Nunes

Programa: Engenharia Elétrica

Este trabalho propõe novas soluções para o Controle Adaptativo Multivariável por Modelo de Referência (MIMO MRAC). As técnicas são aplicáveis a plantas de grau relativo arbitrário e não exigem condições de simetria relacionadas ao ganho de alta frequência (HFG), considerado desconhecido. Controladores robustos com garantia de desempenho transitório são obtidos sem a necessidade de parametrização aumentada do controlador.

Em comparação às técnicas convencionais do MRAC, os controladores propostes são aplicáveis a uma classe mais abrangente de sistemas. Em vez da frágil e restritiva condição de simetria, o HFG deve ter autovalores reais e positivos e forma de Jordan diagonal. Algoritmos com alto ganho e projeção paramétrica são propostos a fim de obter controladores mais robustos em relação àqueles baseados apenas em lei do gradiente. O MRAC Binário (BMRAC) é generalizado para grau relativo arbitrário utilizando diferenciadores exatos, tal que se obtem rastreamento global e exato de saída com garantias de desempenho transitório. Propõe-se ainda uma nova extensão ao BMRAC utilizando adaptação com projeção e alto ganho além de filtragem do sinal de controle e malha de predição. O controlador resultante, chamado BMRAC Estendido, permite rastreamento prático de saída com garantias de transitório. Simulações numéricas são apresentadas a fim de confirmar os resultados teóricos e ilustrar a performance dos controladores.

Abstract of Thesis presented to COPPE/UFRJ as a partial fulfillment of the requirements for the degree of Doctor of Science (D.Sc.)

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Andrei Giordano Holanda Battistel

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Advisors: Liu Hsu
Eduardo Vieira Leão Nunes

Department: Electrical Engineering

This work proposes new solutions to Multivariable Model Reference Adaptive Control (MIMO-MRAC) for plants with arbitrary relative degree without requiring stringent symmetry assumptions related with the plant uncertain high frequency gain (HFG). Robust controllers with guaranteed transient performance are obtained without the need of augmented control parametrization.

Compared to conventional MRAC techniques, the proposed controllers are applicable to a wider class of plants. Instead of fragile symmetry conditions, the HFG is required to have diagonal Jordan form with real and positive eigenvalues, which is a robust and generic condition. Controllers based on parameter projection with high adaptation gain are proposed to obtain improved robustness in comparison to pure gradient adaptive laws. The Binary-MRAC(BMRAC) is generalized to arbitrary relative degree using exact differentiators, such that global exact output tracking is obtained with guaranteed transient properties. A further extension of the BMRAC is proposed, which inherits its high gain and adaptation laws with projection combined with input filtering and prediction loop.The result is the Extended BMRAC (eBMRAC), a robust global practical output tracking controller with guaranteed transient. Simulations are presented to illustrate theoretical developments and performance of the proposed techniques.

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List of abbreviations

SISO Single-Input, Single-Output

MIMO Multi-Input, Multi-Ouput

HFG High Frequency Gain

PR Positive Real

SPR Strictly Positive Real

PDJ Positive Diagonal Jordan Form (with positive and real eigenvalues)

SPD Symmetric Positive Definite

WSPR W-Strictly Positive Real

WASPR W-Almost-Positive Real

MRC Model Reference Control

MRAC Model Reference Adaptive Control

BMRAC Binary Model Referene Adaptive Control

eBMRAC Extended Binary Model Referene Adaptive Control

SSC Smooth Sliding Control

L1-AC L1 Adaptive Control

RED Robust Exact Differentiator

GRED Global Robust Exact Differentiator

Chapter 1

Introduction

In several control applications, the knowledge of system models and/or parameters is not accurate enough. Also, it is often necessary to deal with plants with uncertainties and unmodeled dynamics, or even systems whose parameters vary with time or according to its operating point. In such cases, it is not unusual that fixed-gain controllers do not suffice for a desirable operation. Such applications brought up the need to develop strategies to design controllers that would adjust themselves recursively during the system operation. This adjustment would be done based both on prior system knowledge and on information obtained through measurements, such as input-output or states. In other words, the desired controller would "adapt" itself over time based on some form of system identification [1, 2].

The term "Adaptive Control" have been used since the 1950s, initially motivated by the problem of designing autopilots for aircrafts operating at a wide range of speeds and altitudes [1]. Since aircraft dynamics are conceptually nonlinear and time varying, a single fixed-gain controller would not suffice due to large parameter variations. This lead to adoption of gain scheduling, based on the fact that aircraft dynamics can be approximated by a linear model for a given operating point specified by speed and altitude [2].

Several strategies to self-adjusting controllers were proposed thereafter, such as rudimentary model reference schemes [3],[4], sensitivity rules [5],[6] and the MIT rule. Later on, Kalman proposed a self-tunning optimal linear quadratic controller which was updated based on explicit identification of the system parameters [7].

The 1960s brought advances and theoretical understanding through Lyapunov analysis and the use of state variable representation. On the other hand, the crash of the X-15 test plane was partially attributed to the adaptive system [8], which eventually led to some loss of interest in the field. However, some important contributions were made, such as the development of Dynamic Programming and the advances in System Identification. Also, previously proposed techniques were revisited in the light of the Lyapunov method [9], providing better understanding and restating the importance of rigorous theoretical analysis.

Significant amount of research regarding stability proofs and conditions succeeded such as [10]. In 1979, Egardt [11] showed that adaptive strategies would become unstable in the presence of disturbances. The works by Rohrs[12] and Ioannou [13], brought more examples of the lack of robustness which raised awareness for the fact that the assumptions under which stability was guaranteed was very sensitive to unmodeled dynamics, triggering the interest on Robust Adaptive Control.

Regarding the multi-input multi-output (MIMO) systems, several techniques were later proposed, when the ideia of model reference (or model following) was already discussed, such as [14], [15], [16], [10], [17] and [18]. Significant progress was achieved through [19] where the relative degree for MIMO systems was introduced, as well as the concept of interactor matrix.

Adaptive Control is nowadays a mature branch of control theory with a rich and vast literature, including several textbooks as [20], [21], [2],[22] among others. It is also a field of current interest in diverse engineering and science fields, with open research problems [23].

In the specific case of MIMO systems, there are both theoretical challenges and practical interest, since a variety of systems have multiple inputs and multiple outputs. Also, some concepts known for the scalar case have to be reexamined and generalized for multivariable systems.

Several works proposed solutions to the MIMO MRAC problem. An early design for unknown HFG was proposed assuming that a symmetryzing matrix S_p is known such that $K_pS_p = (K_pS_p)^T > 0$ is known [24], [21], [2]. This is a not only restrictive requirement when K_p is unknown, but also a fragile condition that is easily destroyed for arbitrarily small parametric disturbances.

Circumvention of the symmetry requirements were later possible by using factorization of K_p such as [25], which led to controller overparametrization. A recently proposed approach based on generalized passivity resulted in a controller that does not involve augmented parametrization and also exempts the symmetry assumption. However, the drawback is that it is restricted to uniform relative degree one.

The idea of using variable structures in MRAC is also used in MIMO systems, such as [26], [27], [28] and [29]. Even though the HFG needs to be only anti-Hurwitz, not symmetric, the Sliding Mode Control (SMC) has significant drawbacks such as chattering and sensitivity to noise. This encouraged the use of adaptation laws with projection or sigma-modifications, such that parameters are limited. In techniques such as the BMRAC, the gain can be tuned up to a high value to improve performance and still result in chattering-free control laws [30].

The purpose of this work is to present new techniques for direct Model Reference Adaptive Control (MRAC) using output feedback. In the Direct Adaptive

Control, the controller parameters are updated directly from an adaptive law, whilst the Indirect Adaptive Control the plant parameters are firstly estimated and these estimates are used to calculate a controller. The term output feedback means that state variables are not measured nor used for control purposes.

This work is dedicated to propose new MRAC techniques that require less restrictive assumptions than those found in the literature for MIMO systems. Also, we are concerned in obtaining robust controllers with guaranteed transient performance.

We propose a new MRAC algorithm that is similar to conventional designs, such as seen in [21],[2]. Its advantages are the relative simplicity, since it is fairly related to standard designs; the applicability to plants with arbitrary relative degree and the exemption of symmetry requirements, such that it can be applied to a wider class of plants in comparison to conventional techniques.

It is known that pure gradient adaptive control may suffer from lack of robustness and poor adaptation transient. This has motivated us to propose an extension to a recently proposed Binary MRAC (BMRAC) technique [31]. This algorithm is the MIMO extension to a projection based controller [32] which assures boundedness on controller parameters. The resulting controller is shown to have improved robustness and guaranteed transient performance.

Inspired by the BMRAC technique and in the light of the recently proposed L1 Adaptive Control (L1-AC) [33], which was widely discussed and uses interesting control ingredients, we also propose a further extension to the BMRAC, which we called Extended BMRAC (eBMRAC). This controller is based on a combination of early proposed techniques and shows to present interesting robustness and guaranteed transient properties. Also, we show that it is possible to circumvent fundamental limitations of the L1-AC.

In the next sections we present some basic definitions we use throughout the text and a brief literature review, focusing mainly on the techniques within the scope of this thesis. For further and more general references on adaptive control, see [23].

1.1 Basic Definitions

In this section we present basic definitions, Lemmas and Theorems that are used throughout the text.

1.1.1 Linear Systems

Consider a Linear Time Invariant (LTI) system described by the following realization or state-space model with p inputs and q outputs.

$$\dot{x}(t) = Ax(t) + Bu(t) \tag{1.1}$$

$$y(t) = Cx(t) + Du(t) \tag{1.2}$$

with $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times p}$, $C \in \mathbb{R}^{q\times n}$ and $D \in \mathbb{R}^{q\times p}$; $x \in \mathbb{R}^n$ is the system state, $u \in \mathbb{R}^p$ is the input and $y \in \mathbb{R}^q$ is the output.

This system can be described in input-output form by

$$Y(s) = G(s)U(s) \tag{1.3}$$

$$G(s) = C(sI - A)^{-1}B + D \tag{1.4}$$

where I is the $n \times n$ identity matirx, $Y(s)$ and $U(s)$ are the Laplace transforms of $y(t)$ and $u(t)$ respectively, and $G(s)$ has q rows and p columns and its elements are rational functions of s.

Another way to describe an LTI system is using operators [21]. Consider the inverse Laplace transform of $G(s)$, the output for a given input $u(t)$ is the convolution

$$y(t) = \int_0^t g(t-\tau)u(\tau)d\tau \tag{1.5}$$

Considering $G(s)$ as an operator whose operation is defined by (1.5), it is convenient to express the system (1.3) as

$$y(t) = G(s)[u](t) \tag{1.6}$$

such that $G(s)[u](t)$ denotes the output of a system whose operator representation is $G(s)$ and input is $u(t)$.

The concepts of Controllability and Observability of a system are useful and defined as follows.

Definition 1 (Controllability [34]) *The state equation (1.1) or the pair* (B, A) *is said to be controllable if for any initial state* $x(0) = x_0$ *and any final state* x_1*, there exists an input that transfers* x_0 *to* x_1 *in a finite time. Otherwise (1.1) is said to be uncontrollable.*

Definition 2 (Observability [34]) *The state equation (1.1)–(1.2) is said to be observable if for any unknown initial state* $x(0) = x_0$*, there exists a finite* $t_1 :> 0$ *such that the knowledge of the input* $u(t)$ *and the output* $y(t)$ *over* $[0, t_1]$ *suffices* *to determine uniquely the initial state $x(0)$. Otherwise the equation is said to be unobservable.*

It is possible to verify whether a system is Controllable/Observable or not using the following matrices.

$$C_M = \begin{bmatrix} B \; AB \; A^2B \; \dots \; A^{n-1}B \end{bmatrix} \tag{1.7}$$
$$O_M = \begin{bmatrix} C^T \; A^TC^T \; (A^T)^2C^T \; \dots \; (A^T)^{n-1}C^T \end{bmatrix}^T \tag{1.8}$$

A system is controllable if and only if $rank(C_M) = n$ and is observable if and only if $rank(O_M) = n$. For other necessary and sufficient conditions on Controllability and Observability, see [34].

In this work we are particularly interested in the $o$bservability index, ν, which is the least integer such that

$$O_M = \begin{bmatrix} C^T \; A^TC^T \; (A^T)^2C^T \; \dots \; (A^T)^{\nu-1}C^T \end{bmatrix}^T = n$$

1.1.2 Stability of Dynamic Systems

We use the standard Lyapunov stability definitions. For further reference, the reader is referred to [2, 21, 35].

Consider a system described by the possibly nonlinear differential equation

$$\dot{x} = f(x,t), \quad x(t_0) = x_0 \tag{1.9}$$

where $x(t) \in R^n$, $f : \mathcal{J} \times \mathcal{B}(r) \to \mathcal{R}$, $\mathcal{J} = [t_0, \infty)$ and $\mathcal{B}(r) = \{x \in \mathbb{R}^n | |x| < r\}$. We assume that f possesses one and only solution $x(t) = x(t; x_0, t_0)$ for every $x_0 \in \mathcal{B}(r)$ and every $t_0 \in \mathbb{R}^+$.

Assume the system (1.9) has a solution $x(t) = x(t; x_0, t_0)$ for a given pair of $\{x_0, t_0\}$. The condition $f(0,t) = 0, \forall t \geq t_0$ means that the origin $x_e = 0$ is an equilibrium state of the system. If there exists $h > 0$ such that the ball $B(h) = \{\in \mathbb{R}^n : \|x\| \leq h\}$ contains no other equilibrium states, $x_e = 0$ is said to be an isolated equilibrium state.

Definition 3 (Stability [2]) *The equilibrium state $x_e = 0$ of system (1.9) is stable if for every $\epsilon > 0$ and any $t_0 \geq 0$ there exists a $\delta(\epsilon, t_0) > 0$ such that $\|x_0\| < \delta$ implies that $\|x(t; x_0, t_0)\| < \epsilon$, $\forall t \geq t_0$. The equilibrium state $x_e = 0$ is unstable if it is not stable.*

Definition 4 (Uniform Stability [2]) *The equilibrium state $x_e = 0$ of system (1.9) is uniformly stable if $\delta(\epsilon, t_0) = \delta(\epsilon)$ in Definition 3*

Definition 5 (Attractive Equilibrium [2]) *The equilibrium state $x_e = 0$ of system (1.9) is attractive if for every $t_0 \geq 0$ there exists a $\rho(t_0) > 0$ such that $\|x_0\| < \rho$ implies that $\lim_{t\to\infty} x(t; x_0, t_0) = 0$.*

Definition 6 (Uniform Asymptotic Stability [2]) *The equilibrium state $x_e = 0$ of system (1.9) is uniformly asymptotically stable if it is uniformly stable and if for some $\delta_1 > 0$, every $\epsilon_1 > 0$, and any $t_0 \geq 0$ there exists a $T(\epsilon_1, \delta_1) > 0$ such that $\|x_0\| < \delta_1$ imples that $\|x(t; x_0, t_0)\| < \epsilon_1$, $\forall t \geq t_0 + T$.*

We are also interested in systems whose trajectories are attracted to a small vicinity of the equilibrium point. We shall refer to *practical stability* according to the following definition.

Definition 7 (Practical Stability [36], [37]) *The system $\dot{x} = f(t, x, v)$ is said to be uniformly input-to-state practically stable (ISpS) if there existe $\beta \in \mathcal{KL}, \gamma \in \mathcal{K}$, referred as the $ISpS$ gain, and a nonnegative constant κ, such that for each initial time $t_0 \geq 0$ and initial condition $x(t_0)$, and each bounded measurable input signal $v(\cdot)$ defined on $[\ t_0\infty\ )]$, the solution exists on $[\ t_0\infty\ )]$ and is such that*

$$\|x(t)\| \leq \beta(\|x(t_0)\|, t - t_0) + \gamma(\left\|v_{[t_0,t]}\right\|_\infty) + \kappa \tag{1.10}$$

Where $\left\|v_{[t_0,t]}\right\|_\infty = sup_{t\in[t_0,t]}|f(t)|$. When (1.10) is satisfied with $v \equiv 0$, we say that the system is uniformly globally asymptotically practically stable (GApS), and in addition, if $\beta = ce^{-a(t-t_0)|x(t_0)|}$, wehre c and a are generic positive constants, it is said to be uniformly globally exponentially practically stable (GEpS)

1.1.3 Stable Polynomials

Stability of linear systems described in state-space form (1.1) or input-output (1.3) is directly related to the eigenvalues of the matrix A or the poles of $G(s)$, given by the roots of the characteristic equation $det(sI - A) = 0$.

Consider the polynomial $X(s) = \alpha_n s^n + \alpha_{n-1} s^{n-1} + \ldots + \alpha_0$.

Definition 8 *[2] We say that $X(s)$ is* **monic** *if $\alpha_n = 1$ and the* **degree** *of $X(s)$ is n if the coefficient α_n of s^n is nonzero.*

Definition 9 (Hurwitz Polynomial [2]) *A polynomial is said to be* **Hurwitz** *if all the roots of $X(s) = 0$ are located in $Re[s] < 0$*

Definition 10 (Schur Polynomial [2]) *A polynomial is said to be* **Schur** *if all the roots of $X(s) = 0$ are located in $Re[s] < |1|$*

A polynomial is said to be stable if it is Hurwitz for continuous-time systems and Schur for discrete-time systems.

Definition 11 (Hurwitz and Anti-Hurwitz Matrices) *A matrix A is said to be* **Hurwitz** *if all its eigenvalues are negative, that is, the roots of $\lambda I - A = 0$ are in $Re[\lambda] < 0$. Similarly, A matrix A is said to be* **Anti-Hurwitz** *if all its eigenvalues are positive, that is, the roots of $\lambda I - A = 0$ are in $Re[\lambda] > 0$.*

1.1.4 Interactor Matrix and the High Frequency Gain

Consider a continuous-time linear time-invariant (LTI) plant with M inputs and M outputs.

$$y(t) = G(s)[u](t) \tag{1.11}$$

where $y(t) \in \mathbb{R}^M$ is the plant output, $u(t) \in \mathbb{R}^M$ is the plant input, with $t \in [0, \infty)$.

The concept of interactor matrix [38] is important in MRAC schemes, since it determines the system structure at infinity. For design purposes we shall also refer to a modified interactor, which is defined next [2, 21].

Proposition 1 *[21] For any $M \times M$ proper rational full rank transfer matrix $G(s)$ there exists a unique lower triangular polynomial matrix $\xi(s)$, defined as the left interactor matrix of $G(s)$, of the form*

$$\xi(s) = \begin{bmatrix} s^{l_1} & 0 & \dots & \dots & 0 \\ s^{l_1} h_{21}(s) & s^{l_2} & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ s^{l_1} h_{M1}(s) & s^{l_1} h_{M2}(s) & \dots & s^{l_{M-1}} h_{MM-1}(s) & s^{l_M} \end{bmatrix}, \tag{1.12}$$

where $h_{ij}(s)$, $j = 1, \dots, M-1$, $i = 2, \dots, M$, are polynomials divisible by s and $l_i \geq 0$, $i = 1, \dots, M$, are integers such that

$$\lim_{s \to \infty} \xi(s) G(s) = K_p^0$$

is finite and nonsingular

Note that the inverse of the left interactor is not stable. Since in the design of model reference control $\xi^{-1}(s)$ is required to be stable, it is convenient to employ a modified left interactor matrix that has a stable inverse.

Proposition 2 *[21] For any $M \times M$ proper rational full rank transfer matrix $G(s)$ there exists a lower triangular polynomial matrix $\xi(s)$, defined as the modified left interactor (MLI) matrix of $G(s)$, of the form*

$$\xi(s) = \begin{bmatrix} d_1 & 0 & \dots & \dots & 0 \\ h^m_{21}(s) & d_2(s) & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ h_{M1}(s) & h_{M2}(s) & \dots & h_{MM-1}(s) & d^M(s) \end{bmatrix}, \tag{1.13}$$

where $h^m_{ij}(s)$, $j = 1, \dots, M-1$, $i = 2, \dots, M$, are some polynomials and $d_i(s) = s^{l_i} + a^i_1 s^{l_i - 1} + \dots + a^i_{l_i}$, and $i = 1, \dots, M$ are any chosen monic stable polynomials such that

$$\lim_{s \to \infty} \xi(s) G(s) = K_p \tag{1.14}$$

is finite and nonsingular. Furthermore, $h^m_{ij}(s)$ can be chosen as $h^m_{ij}(s)$ = polynomial part of $s^{l_j}\left(h_{ij}(s) + a^i_1 s^{-1} h_{ij}(s) + \dots + a^i_{l_i} s^{-l_i} h_{ij}(s)\right)$ for $j = 1, \dots, M-1$, $i = 2, \dots, M$. For this choice of $h^m_{ij}(s)$, we have

$$\lim_{s \to \infty} \xi(s) G_0(s) = K^0_p \tag{1.15}$$

For proofs, see [21][pp.385].

Thus, Eq. (1.14) defines the High Frequency Gain (HFG) of the system transfer function (matrix) $G(s)$. In the SISO case, k_p is a scalar and the interactor matrix is any stable polynomial of degree $n - m$. The condition usually assumed for adaptive control is that there is prior knowledge on the signal of k_p [].

In the multivariable case, different knowledge and conditions are required on K_p to different adaptive control techniques. In this Thesis we draw special attention to mitigating symmetry requirements on the HFG and seek to use a more robust condition referred to as the PDJ condition (Positive Diagonal Jordan), defined as follows.

Definition 12 (Positive Diagonal Jordan Condition (PDJ)) *A matrix is said to be PDJ if its eigenvalues are real and positive and its Jordan form is diagonal.*

A matrix satisfying the PDJ condition can be multiplied by a symmetric positive definite (SPD) matrix such that the product is also SPD. This result is of particular importance to this work and is stated in the following lemma.

Lemma 1 ([39], [40]) *Given $K_p \in \mathbb{R}^{M \times M}$, there exists $W = W^T > 0$, $W \in \mathbb{R}^{M \times M}$ such that*

$$W K_p = K_p^T W > 0,$$

if and only if K_p is PDJ .

Proof: see [40]. ■

Further, it can be shown that if the leading minors of K_p are nonzero, there exists a matrix $\bar{L}$ such that $\bar{L}K_p$ is PDJ. Consider the LDU factorization [41] of K_p.

$$K_p = L_p D_p U_p$$

where L_p is unit lower triangular, D_p is diagonal and U_p is unit upper triangular. It is possible to choose a diagonal matrix D_0 with positive real and distint eigenvalues such that there is a lower triangular matrix

$$\bar{L} = D_0(L_p D_p)^{-1}$$

This implies that

$$\bar{K}_p = \bar{L}K_p = D_0(L_p D_p)^{-1}(L_p D_p)U_p = D_0 U_p$$

is upper triangular and PDJ. According to Lemma 1, there is W such that $W\bar{L}K_p$ is SPD.

This result is also important to our work, since it shows that even if the plant HFG is not PDJ, a multiplier can be used to satisfty the PDJ condition. This idea is explored in Chapters 2 and 3.

1.1.5 Passivity Concepts

Definition 13 (Positive Real Transfer Functions [35]) *A $M \times M$ proper rational transfer function matrix $G(s)$ is called positive real if*

- *poles of all elements $G(s)$ are in $Re[s] \leq 0$,*
- *for all real ω for which $j\omega$ is not a pole of any element of $G(s)$, the matrix $G(j\omega) + G^T(-j\omega)$ is positive semidefinite, and*
- *any pure imaginary pole $j\omega$ of any element of $G(s)$ is a simple pole and the residue matrix $\lim_{s\to j\omega)}(s - j\omega)G(s)$ is positive semidefinite Hermitian.*

The transfer function $G(s)$ is called strictly postive real if $G(s - \varepsilon)$ is positive real for some $\varepsilon > 0$

Lemma 2 (Positive Real Lemma [35]) *Let $G(s) = C(sI - A)^{-1}B$ be a $M \times M$ strictly proper transfer function matrix where (A, B) is controllable and (A, C) is* *observable. Then $G(s)$ is positive real if and only if there exit matrices $P = P^T > 0$, and $Q = Q^T \geq 0$ such that*

$$A^T P + PA = -Q\,, \tag{1.16}$$

$$PB = C^T\,. \tag{1.17}$$

Lemma 3 (Kalman-Yakubovich-Popov [35]) *Let $G(s) = C(sI - A)^{-1}B$ be a $M \times M$ strictly proper transfer function matrix where (A, B) is controllable and (A, C) is observable. Then $G(s)$ is strictly positive real if and only if there exit matrices $P = P^T > 0$, and $Q = Q^T \geq 0$ and a positive constant ϵ such that*

$$A^T P + PA = -Q - \epsilon P\,, \tag{1.18}$$

$$PB = C^T\,. \tag{1.19}$$

1.2 Problem Description - Multi-Input Multi-Output (MIMO) Case

Similarly to the approach of [21], we will consider simultaneously both continuous-time (CT) systems and discrete-time (DT) systems. To this end, the symbol D is used, in the CT case, as the Laplace transform variable or the time differential operator $D[x](t) = \dot{x}(t), t \in [0, \infty)$, as the case may be or, in the DT case, as the z-transform or the time advance operator $D[x](t) = x(t+1), t \in \{0, 1, 2, 3, ...\}$, as the case may be. A polynomial in D is said to be stable if its roots have strictly negative real parts (Hurwitz polynomial), in the CT case, or are strictly inside the unit circle of the complex plane (Schur polynomial), in the DT case.

Consider an uncertain MIMO LTI plant with M inputs and M outputs described in state space form as

$$D[x_p] = A_p x_p + B_p u\,, \quad y = H_p x_p\,, \tag{1.20}$$

where $x_p(t) \in \mathbb{R}^n$ is the state, $y(t) \in R^M$ is the plant output, $u(t) \in R^M$ is the plant input, with $t \in [0, \infty)$ for the continuous-time (CT) case and $t \in \{0, 1, 2, 3, ...\}$ for the discrete-time (DT) case, and A_p, B_p, H_p are constant uncertain matrices.

In input/output form one has

$$y(t) = G(D)[u](t). \tag{1.21}$$

The objective of the model reference adaptive control (MRAC) can be stated as follows. Given a reference model system of the form:

$$y_m(t) = W_m(D)[r](t), \tag{1.22}$$

where $W_m(D)$ is and $M \times M$ rational transfer function matrix, $y_m(t) \in R^M$ is the reference output, and $r(t) \in R^M$ is an external input signal, find a feedback control signal $u(t)$ for the plant (1.21) with unknown $G(D)$ such that $y(t)$ tracks $y_m(t)$ as close as possible and the closed-loop system is globally stable in the sense that all signals in the system are bounded for any bounded initial conditions and input signals.

The following assumptions are made:

(A1) $G(D)$ has full rank and all its zeros are stable;

(A2) The plant is controllable and observable;

(A3) The observability index ν of $G(D)$, or an upper bound of ν, is known;

(A4) There exists a known diagonal polynomial matrix $\xi_m(D)$, defined as the modified left interactor (MLI) matrix of $G(D)$ of the form $\xi_m(D) = diag\{d_1(D),\ d_2(D),\ \ldots,\ d_M(D)\}$ where $d_i(D)$ are monic stable polynomials of degrees $l_i > 0$, such that the high frequency gain matrix of $G(D)$, defined as $K_p = \lim_{D\to\infty} \xi_m(D)G(D)$. is finite and nonsingular.

The MLI is defined as in Subsection 1.1.4, rewritten to encompass both CT and DT systems as in [21][Lemma 9.1]

$$\xi_m(D) = \begin{bmatrix} d_1(d) & 0 & \ldots & \ldots & 0 \\ h^m_{21}(D) & d_2(D) & 0 & \ldots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ h_{M1}(D) & h_{M2}(D) & \ldots & h_{MM-1}(D) & d^M(D) \end{bmatrix}, \tag{1.23}$$

Note that Assumption (A4) means that the vector relative degree of $G(D)$ is known.

Let the reference signal y_m be generated by the following reference model:

$$y_m = W_m(D)\, r\,; \qquad r, y_m \in \mathbb{R}^M\,, \tag{1.24}$$

The tracking error is then given by

$$e = y - y_m \tag{1.25}$$

The transfer function matrix $W_m(D)$ has the same vector relative degree as $G(D)$. Without loss of generality, it is possible to choose $W_m(D) = \xi_m^{-1}(D)$.

When the plant is known, a control law which achieves the matching between the closed-loop transfer function matrix and $W_m(D)$ is given by

$$u^* = {\theta^*}^T \omega \tag{1.26}$$

where the parameter matrix is written as

$$\theta^* = \left[{\theta_1^*}^T \; {\theta_2^*}^T \; {\theta_3^*}^T \; {\theta_4^*}^T \right]^T , \tag{1.27}$$

with $\theta_1^*, \theta_2^* \in \mathrm{I\!R}^{M(\nu-1)\times M}$, $\theta_3^*, \theta_4^* \in \mathrm{I\!R}^{M\times M}$ and the regressor vector

$$\omega = [\omega_u^T \; \omega_y^T \; y^T \; r^T]^T, \quad w_u, w_y \in \mathrm{I\!R}^{M(\nu-1)} \tag{1.28}$$

is obtained from I/O state variable filters given by:

$$\omega_u = A(D)\Lambda^{-1}(D)u\,, \quad \omega_y = A(D)\Lambda^{-1}(D)y\,, \tag{1.29}$$

where $A(D) = [ID^{\nu-2} \quad ID^{\nu-3} \quad \cdots \quad ID \quad I]^T$, $\Lambda(D) = \lambda(D)I$ with $\lambda(D)$ being a monic stable polynomial of degree $\nu - 1$.

The plant transfer matrix can be expresses as a product $G(D) = Z_0(D)P_0^{-1}(D)$. With the matching control of Eq. (1.26), the parameter matrix (1.27) and the regressor (1.28), the matching equation

$$\theta_1^{*T} A(D)P_0(D) + \left(\theta_2^{*T} A(D) + \theta_3^{*T}\Lambda(D)\right) Z_0(D) = \Lambda(D) \left(P_0(D) - \theta_4^{*T}\xi_m(D)\right) \tag{1.30}$$

defines the matching parameters $\theta_1^{*T}, \theta_2^{*T}\theta_3^{*T}$, and θ_4^{*T} [21]. Also, the matching conditions require that $\theta_4^{*T} = K_p^{-1}$.

However, since the plant is unknown, the desired parameters matrix θ^* is also unknown. In this case, the following control law can be used

$$u(t) = \theta(t)\omega(t) \tag{1.31}$$

It is important to note that the in the MIMO case, solution of matching equation is possibly non-unique, such that Eq. (1.30) may admit a set of solutions to θ^*. This is worthy of attention since in this case it is not possible to identify the plant even with a rich signal[42]. In this work we are interested in boundedness of the closed loop signals and tracking of the reference model. The problem of parameter convergence is discussed in detail in [43] and [42].

An error equation can be developed extending the usual approach for SISO

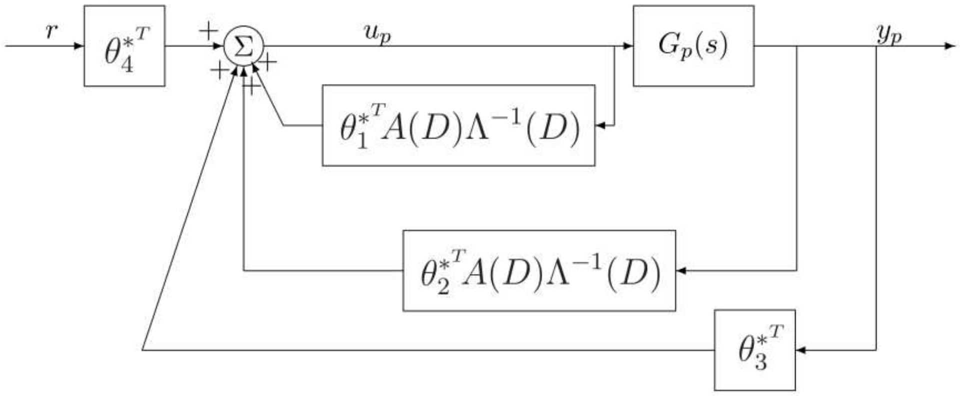

Figure 1.1: Block Diagram of MRAC structure

MRAC to the multivariable case [21].

Defining the state vector $X = \left[x_p^T,\ \omega_u^T,\ \omega_y^T\right]^T$ with the following dynamics

$$D[X] = A_0X + B_0u. \tag{1.32}$$

Then, adding and subtracting B_0u^* and

$$D[X] = A_0X + B_0u + B_0\left(\theta^{*T}\omega_r + K_\theta^{*T}r\right) - B_0u^*. \tag{1.33}$$

noting that there are matrices Ω_1 and Ω_2 such that $\omega = \Omega_1X + \Omega_2r$, it follows that

$$D[X] = A_cX + B_cK_p\left[u - u^*\right] + B_cr, \quad y = H_0X \tag{1.34}$$

with $A_c = A_0 + B_0\theta^{*T}\Omega_1,\ B_c = B_0K_\theta^{*T} = B_0K_p^{-1}$. The reference model can be described by

$$D[X_m] = A_cX_m + B_cr \tag{1.35}$$

The error state x_e dynamics is given by

$$D[x_e] = A_c\,x_e + B_cK_p[u - {\theta^*}^T\omega], \qquad e = H_o\,x_e, \tag{1.36}$$

$\{A_c, B_c, H_o\}$ is a nonminimal realization of $W_m(D)$, so that the error equation can be written in input-output form as

$$e = W_m(D)K_p\left[u - {\theta^*}^T\omega\right]. \tag{1.37}$$

1.3 Multivariable Model Reference Adaptive Control and its Symmetry Conditions

The conventional multivariable direct Model Reference Adaptive Control (MIMO MRAC) algorithm is designed under the stringent assumption that a multiplier S_p for the High Frequency Gain (HFG) matrix K_p is known, such that S_pK_p becomes symmetric positive definite (SPD) [2, 21]. This assumption is nongeneric, thus fragile, since symmetry is easily destroyed by an arbitrarily small parametric perturbation on K_p.

To illustrate this issue, we consider the case of direct adaptive visual tracking for planar manipulators using a fixed camera (plant) with optical axis orthogonal to the robot workspace. The camera orientation is uncertain with respect to the robot workspace coordinates [44, 45]. The objective is to control the robot so that the image of its end-effector tracks a desired trajectory in the image plane. This problem is discussed in detail in Subsection 2.3.2.

In this case, the HFG is essentially a rotation matrix where ϕ is the angle between camera and manipulator frames.

$$K_p = \begin{bmatrix} cos(\phi) & -sin(\phi) \\ sin(\phi) & cos(\phi) \end{bmatrix} \tag{1.38}$$

Except for trivial cases, this matrix is not SPD and has complex eigenvalues. Considering a nominal $\phi_{nom} = 45°$, K_p is:

$$K_{p_{nom}} = \begin{bmatrix} 0.7071 & -0.7071 \\ 0.7071 & 0.7071 \end{bmatrix}$$

A possible symmetrizig matrix would be

$$S_p = \begin{bmatrix} 2 & 1 \\ -2 & 1 \end{bmatrix}$$

such that S_pK_p is SPD.

$$S_pK_{p_{nom}} = \begin{bmatrix} 2.1213 & -0.7071 \\ -0.7071 & 2.1213 \end{bmatrix}$$

However, if the actual value of ϕ is slightly different of ϕ_{nom}, say $\phi = 45.01°$, the product S_pK_p is no longer SPD.

$$S_pK_p = \begin{bmatrix} 2.1212 & -0.7075 \\ -0.7067 & 2.1214 \end{bmatrix}$$

Recent progress in circumventing the symmetry assumption has brought renewed attention to the MIMO MRAC problem. By using matrix factorization methods, it is possible to design stable MIMO-MRAC for plants with uncertain and possibly non-symmetric HFG [21, 25, 46–50]. However, while such factorization approach represents a quite general solution, it leads to controller overparametrization.

Further details on the method based on SDU factorization is given in Subsection 1.4.2. The necessity of overparametrization not only increases the number of adapted parameters by a scale of $M(M-1)/2$ as it can lead to loss of robustness, since the matching parameters form a linear manifold (and therefore unbounded) [51, 52].

For plants with uniform relative degree one, using the Immersion and Invariance approach (I & I) [53], the symmetry condition was also circumvented. The I & I method requires the knowledge of a less restrictive multiplier Γ such that $K_p\Gamma^T + \Gamma K_p^T > 0$ [54]. However, the adaptive law had to include additional filtering and auxiliary control signals. Another possibility for the case of relative degree one appeared more recently in [39] and [44] based on a generalized passivity condition named WSPR, instead of the usual SPR (Strictly Positive Real) condition. The concept of WASPR (W-almost SPR) was also introduced in [39] and consists in requiring the plant to become WSPR through some static output feedback. It was shown in [44] that a necessary and sufficient condition for minimum phase plants of relative degree one to be WASPR is that the HFG matrix has a positive diagonal Jordan form. We shall refer to this condition as the PDJ condition. The WSPR approach leads to a simpler adaptation law which is just the conventional (unnormalized) Lyapunov-based law without the augmented parametrization required in the factorization methods.

Recalling the visual servoing example, we note that K_p in Eq. 1.38 is not PDJ either since it has complex eigenvalues. However according to the results of Subsection 1.1.4, we know that it is possible to obtain a multiplier $\bar{L}$ that guarantees that $\bar{L}K_p$ does satisfy the PDJ condition. Considering the nominal value of $\phi_{nom} = 45°$, it is possible to use the multipler $\bar{L}$ as

$$\bar{L} = \begin{bmatrix} 35.3553 & 0 \\ -0.7071 & 0.7071 \end{bmatrix}$$

such that $\bar{L}K_p$ satisfies the PDJ condition in the interval $-71° < \phi < 73°$. Unlike the symmetry condition, which can be easily destroyed by small parametric perturbation, PDJ condition is valid on a set. The visual servoing application is discussed in detail in Subsection. 2.3.2

In the next section we present a brief review of related MIMO MRAC designs to provide a better comparison with the new techniques to be proposed in the next chapters. These techniques require the PDJ condition on K_p and mitigate the need of symmetry conditions.

1.4 Review of Previous Designs

Only CT systems will be considered in this section. The Lyapunov design develops the adaptation law from a Lyapunov function involving the error state and the parametric errors.

For simplicity, consider the case of *uniform relative degree one*. In the scalar case, it is well known that a simple way to form the Lyapunov function is by choosing an SPR reference model so that the positive real lemma can be applied to the error state equations. In the multivariable case, the transfer function $W_m(D)K_p$ can only be SPR if K_p is symmetric. In order to relax this stringent condition, two recent methods can be used: (a) HFG factorizations with corresponding control reparametrization [25] and (b) a generalized passivity condition (WSPR) [39]. We briefly overview these methods in order to put into perspective our proposed methods.

1.4.1 Lyapunov Design

Let us recall the main steps in the SISO design:

1. $W_m(s)$ is a scalar transfer function chosen to be SPR.

2. The adaptive control law is

$$u = \theta^T \omega \,, \tag{1.39}$$

   where $\theta(t)$ is an estimate of θ^*. With (1.39) the output error is a linear function of the parameter error $\tilde{\theta} = \theta - \theta^*$,

$$e = W_m(s)K_p \left[\tilde{\theta}^T \omega\right] . \tag{1.40}$$

3. Assuming that $sign(K_p)$ is known, stability and convergence of $e(t)$ are assured by the update law (SISO case)

$$\dot{\theta} = -\gamma \; sign(K_p)\omega e \,. \tag{1.41}$$

The MIMO generalizations of these steps, discussed in the textbooks [2, 21], can be briefly summarized as follows.

For Step 1 ,a diagonal reference model $W_m(s)$ is selected as in (1.24). For Step 2, the matrix version of the control law (1.39) is employed, where θ is a matrix, while ω is an enlarged vector. The MIMO error equation retains the same form of the SISO error equation (1.40), except that $W_m(s)$, K_p and $\tilde{\theta}$ are matrices. For Step 3, different assumptions about K_p have been made. An early design for unknown K_p was considered in [2, 21, 24], under the additional assumption that a matrix S_p is known such that $K_pS_p = (K_pS_p)^T > 0$.

This assumption is quite restrictive when K_p is unknown. However, it is illustrative to show the corresponding Lyapunov function and adaptive law of this pioneering method [24]. To this end, consider a state realization of the error equation (1.40):

$$\dot{x}_e = A_c\, x_e + B_c K_p[\tilde{\theta}^T \omega], \qquad e = H_o\, x_e, \tag{1.42}$$

where $\{A_c, B_c, H_o\}$ is a realization of $W_m(s)$. As usual, assume that $W_m(s)$ is SPR. Then, by the positive real lemma, for some symmetric positive definite (SPD) P and Q one has $A_c^T P + P A_c^T = -Q; \quad PB = H_o^T$. The Lyapunov function is given by

$$V = x_e^T P x_e + tr(\tilde{\theta}\Gamma_p \tilde{\theta}^T) \tag{1.43}$$

where $\Gamma_p = K_p^T S_p^{-1} = (S_p^{-1})^T (K_pS_p)^T (S_p^{-1}),\ P = P^T > 0$. Differentiating with respect to t, one gets

$$\dot{V} = x_e^T(A_c^T P + P A_c)x_e + 2(\omega^T \tilde{\theta} K_p^T B_c^T P x_e + tr(\tilde{\theta}\Gamma_p \dot{\tilde{\theta}}^T)) \tag{1.44}$$

Then the following type of adaptive law was proposed in [24]:

$$\dot{\tilde{\theta}}^T = -S_p e \omega^T; \tag{1.45}$$

From (1.44) and (1.45) one gets

$$\dot{V} = -x_e^T Q x_e + 2(\omega^T \tilde{\theta} K_p^T B_c^T P x_e - tr(\tilde{\theta}\Gamma_p S_p e \omega^T)) \tag{1.46}$$

or (Note: if $x,\ y \in \mathbb{R}^n$ then $tr(xy^T) = y^T x$),

$$\dot{V} = -x_e^T Q x_e + 2(\omega^T \tilde{\theta} K_p^T B_c^T P x_e - \omega^T \tilde{\theta}\Gamma_p S_p e) \tag{1.47}$$

For negative semi-definitness, we set $\Gamma_p S_p = K_p^T$. This means that S_p must be chosen such that $K_P S_P = (K_P S_P)^T$ becomes SPD. Thus, a quite stringent prior knowledge (of one such S_p) is required about K_p.

1.4.2 Gain factorization

A new parametrization can be performed using a factorization of the high frequency gain K_p. For this, we need the following lemma (see [25])

Lemma 4 *Every $m \times m$ real matrix K_p with nonzero leading principal minors $\Delta_1,\ \Delta_2, \cdots, \Delta_m$ can be factored as*

$$K_p = SDU\,, \tag{1.48}$$

where S is symmetric positive definite, D is diagonal, and U is unity upper triangular.

Proof. Since the leading principal minors of K_p are nonzero, there exists a unique factorization

$$K_p = L_1 D_p L_2^T\,, \tag{1.49}$$

where L_1 and L_2 are unity lower triangular and

$$D_p = diag\left\{\Delta_1,\ \frac{\Delta_2}{\Delta_1},\ \cdots,\ \frac{\Delta_m}{\Delta_{m-1}}\right\}\,. \tag{1.50}$$

Factoring D_p as

$$D_p = D_+ D\,, \tag{1.51}$$

where D_+ is a diagonal matrix with positive entries, we rewrite (1.49) as $K_p = L_1 D_+ L_1^T L_1^{-T} D L_2^T$, so that (1.48) is satisfied by

$$S = L_1 D_+ L_1^T\,, \qquad\qquad U = D^{-1} L_1^{-T} D L_2^T\,. \tag{1.52}$$

■

Remark. The above factorization $K_p = SDU$ is not unique because the positive diagonal matrix D_+ is a free parameter.

The SDU factorization of K_p is then employed to derive a new form of the error equation. Substituting $K_p = SDU$ in (1.37) we obtain

$$e = W_m(s)SD[Uu - U\theta_1^{*T}\omega_1 - U\theta_2^{*T}\omega_2 - U\theta_3^* y - U\theta_4^* r]\,. \tag{1.53}$$

A further refinement of this expression will make sure that the control law is well-defined. With the decomposition

$$Uu = u - (I - U)u \tag{1.54}$$

where $(I - U)$ is strictly upper triangular, it is possible to define the control signal u as a function of $(I - U)u$. No static loops can appear, because u_1 depends on

$u_2, \cdots, u_m$, while u_2 depends on $u_3, \cdots, u_m$, and so on. The unknown entries of U are incorporated in the new parametrization by defining $K_1 = U\theta_1^{*T}$, $K_2 = U\theta_2^{*T}$, $K_3 = U\theta_3^*$, and $K_4 = U\theta_4^*$, and rewriting (1.53) as

$$e = W_m(s)SD[u - K_1\omega_1 - K_2\omega_2 - K_3 y - K_4 r - (I-U)u]\,. \tag{1.55}$$

Next, new parameter vectors Θ_i^* are introduced via the identity

$$\begin{bmatrix}\Theta_1^{*T}\Omega_1 & \Theta_2^{*T}\Omega_2 & \cdots & \Theta_m^{*T}\Omega_m\end{bmatrix}^T \equiv K_1\omega_1 + K_2\omega_2 + K_3 y + K_4 r + (I-U)u\,. \tag{1.56}$$

In addition to the concatenated i^{th} rows of the matrices K_1, K_2, K_3, K_4, each row vector Θ_i^{*T} includes the unknown entries of the i^{th} row of $(I-U)$. The corresponding regressor vectors are

$$\begin{aligned}\Omega_1^T &= \begin{bmatrix}\omega^T & u_2 & u_3 & \cdots & u_m\end{bmatrix}, \\ \Omega_2^T &= \begin{bmatrix}\omega^T & u_3 & \cdots & u_m\end{bmatrix}, \\ &\vdots \\ \Omega_m^T &= \begin{bmatrix}\omega^T\end{bmatrix}.\end{aligned} \tag{1.57}$$

The error equation (1.55) has thus been brought to the new form

$$e = (W_m(s)S)D\left(u - \begin{bmatrix}\Theta_1^{*T}\Omega_1 & \Theta_2^{*T}\Omega_2 & \cdots & \Theta_m^{*T}\Omega_m\end{bmatrix}^T\right). \tag{1.58}$$

In this new parametrization the adaptive control law is

$$u = \begin{bmatrix}\Theta_1^T\Omega_1 & \Theta_2^T\Omega_2 & \cdots & \Theta_m^T\Omega_m\end{bmatrix}^T, \tag{1.59}$$

where Θ_i are the estimates of Θ_i^*. Compared with the control law (1.31), this control makes use of a larger number of parameters.

The key feature of the error equation (1.58) is that the diagonal matrix D appears in the place of the K_p, and an assumption can be made about the signs of its entries $d_1, \cdots, d_m$. The following lemma holds (see [25]).

Lemma 5 *For any $A = diag\{-a_i\}$, $a_i > 0$, $(i = 1, \cdots, m)$, and any $m \times m$ unity lower triangular matrix L_1, there exists $D_+ = diag\{d_i^+\}$, $d_i^+ > 0$, such that*

$$W_m(s)S = (sI - A)^{-1}L_1 D_+ L_1^T \tag{1.60}$$

is SPR.

Proof. See proof in [55]. ∎

Combining the state $x \in \mathbb{R}^n$ of the plant (1.21) with the filter states ω_1, and ω_2 in CT ($D = s$), it is defined $X = [x^T \;\; \omega_1^T \;\; \omega_2^T]^T \in \mathbb{R}^{n+2m(\nu-1)}$. Where X_M denotes the state of the corresponding nonminimal realization $C_m(sI - A_m)^{-1}B_m$ of $W_m(s)S$ where $C_m B_m = S$. Then, the state error $z = X - X_m$ and the output error e in (1.58) satisfy

$$\begin{aligned}\dot{z} &= A_m z + B_m D \left(u - \begin{bmatrix} \Theta_1^{*T}\Omega_1 & \cdots & \Theta_m^{*T}\Omega_m \end{bmatrix}^T \right), \\ e &= C_m z\,. \end{aligned} \tag{1.61}$$

Because $W_m(s)S$ is SPR, there exist matrices $P_m = P_m^T > 0$ and $Q_m = Q_m^T > 0$ satisfying

$$\begin{aligned} A_m^T P_m + P_m A_m &= -2Q_m\,, \\ P_m B_m &= C_m^T\,. \end{aligned} \tag{1.62}$$

An update law for the control parameters vectors Θ_i is design in the adaptive control (1.59) in a complete analogy with SISO adaptive case. The following Lyapunov function is used

$$V = \frac{1}{2}\left(z^T P_m z + \sum_{i=1}^{M} \gamma_i^{-1} |d_i| \tilde{\Theta}_i^T \tilde{\Theta}_i \right), \tag{1.63}$$

where $\tilde{\Theta}_i = \Theta_i - \Theta_i^*$ are the parameter errors, d_i are the entries of D, and $\gamma_i > 0$ are adaptation gains. The time derivative of (1.63) along the trajectories of the error system (1.61) yields

$$\begin{aligned} \dot{V} = -z^T Q_m z + z^T P_m B_m D \begin{bmatrix} \tilde{\Theta}_1^T \Omega_1 & \cdots & \tilde{\Theta}_M^T \Omega_M \end{bmatrix}^T + \sum_{i=1}^{M} \gamma_i^{-1} |d_i| \tilde{\Theta}_i^T \dot{\tilde{\Theta}}_i \\ = -z^T Q_m z + \sum_{i=1}^{M} \gamma_i^{-1} |d_i| \tilde{\Theta}_i^T \left[\gamma_i \, sign(d_i) e_i \Omega_i + \dot{\tilde{\Theta}}_i \right]. \end{aligned}$$

An update law which renders $\dot{V}$ nonpositive, $\dot{V} = -z^T Q_m z$, is

$$\dot{\Theta}_i = \dot{\tilde{\Theta}}_i = -\gamma_i \, sign(d_i) e_i \Omega_i\,, \quad (i = 1, \cdots, M)\,. \tag{1.64}$$

Thus, the adaptive control (1.59) and the update law (1.64) guarantee $\tilde{\Theta}_i, \Theta_i \in L^\infty$ and $z \in L^\infty \cap L^2$.

Because $z = X - X_M$ and X_M are bounded, X is also bounded and, consequently, y, ω_1 and ω_2 are bounded. Since $r(t)$ is uniformly bounded by assumption, ω is bounded. To prove that $\Omega_1, \cdots, \Omega_m$ are bounded and, hence, u is also bounded, we return to (1.57). The advantage of the structure of (1.57), resulting from the control parametrization, is that $\Omega_m = \omega$ being bounded, implies that $u_m = \Theta_m^T \Omega_m$ is bounded. Therefore $\Omega_{m-1}^T = [\omega^T \quad u_m]$ is bounded. Repeating this argument it is possible to show that $u_{m-1}, \cdots, u_2, u_1$ are all bounded. Therefore, all the signals in the closed-loop system are bounded. This also implies that $\dot{z}$, $\dot{e}$, $\dot{\Theta}_i$ and consequently $\ddot{V}$ are all uniformly bounded. Finally, the usual argument invoking Barbalat's Lemma proves that $z(t), e(t) \to 0$ as $t \to \infty$.

Thus, the result is a controller that assures that all the closed loop signals are uniformly bounded and the tracking error $e(t)$ converges to zero.

1.4.3 Generalized Passivity (WSPR)

The factorization approach leads to a more involved adaptive law as a result of the augmented parametrization. Moreover, it is dependent of the input-output pairing since this affects the signs of the leading principal minors of K_p and also the assumptions that they be nonzero.

In order to circumvent such limitations, possibly at the expense of a more restrictive assumption about the unknown HFG, a new method was developed based on a generalized passivity condition described by a modified positive real lemma which characterizes the so called WSPR systems. The advantage of the resulting adaptation algorithm is its simplicity. It does not involve augmented parametrization, is independent of the input/output pairing and leads to adaptive laws which are very close to the traditional ones.

From the Positive Real Lemma (1.17), one has that $B^T P B = B^T C^T = CB$. Thus, a LTI system (1.1) can only be SPR if ($K_p = CB$) is *symmetric and positive definite* (SPD). This condition is usually not satisfied by real systems. A more relaxed condition is the following. A solution to overcome this difficulty was recently proposed in [39, 44], exploiting the more general concept of passivity associated with the WSPR condition defined hereafter together with the condition of WASPR ("W Almost SPR") and some basic results related with such conditions , where the last can be obtained by multiplying the output error vector by a triangular matrix.

Definition 14 (WSPR [39, 44]) *A linear time-invariant system with state realization $\{A_K, B, C\}$, where $A_K \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$, and $C \in \mathbb{R}^{m\times n}$ is said to be W–Strictly-Passive (WSP) and its transfer function $C(sI - A_K)^{-1}B$ is said W–Strictly Positive Real (WSPR), if there exist symmetric and positive definite matrices P, Q,* *and* W *such that*

$$A_K^T P + P A_K = -Q\,, \tag{1.65}$$

$$PB = C^T W\,. \tag{1.66}$$

■

It was noted in [44] that, unlike the SPR condition, the equations (1.65)-(1.66) no longer require the symmetry condition of CB but instead of $W(CB)$, according to Lemma 1 As in [56],[30], if K_p does not satisfy the PDJ condition, we choose an adequate matrix multiplier L such that LK_p does satisfy such condition in a robust way using the above result.

Definition 15 (WASPR [39, 40]) *A linear time-invariant system with state realization* $\{A, B, C\}$*, is said to be WASPR if it can be made WSPR through a static output feedback, i.e., if there exists* $K \in \mathbb{R}^{m\times m}$ *such that* $C(sI - A_K)^{-1}B$ *results WSPR, with* $A_K = A - BKC$*.* ■

Theorem 1 (WASPR Theorem [40]) *Every strictly proper and minimum phase system* $\{A, B, C\}$*, with* $A \in \mathbb{R}^{n\times n}, B \in \mathbb{R}^{n\times m}, C \in \mathbb{R}^{m\times n}$ *and* $m\times m$ *transfer function* $C(sI - A)^{-1}B$*, can be made WSPR via (sufficiently large) output feedback, if and only if* K_p *has positive real eigenvalues and its Jordan form is diagonal.* ■

The passifying multiplier L is chosen so that the modified system $\{A_K, B_c K_P, LH_0\}$ satisfies the WASPR condition of Theorem 1. This allows us to deal with plants with non-symmetric HFG, requiring it only to satisfy the PDJ condition, which is robust and non fragile in opposition to symmetry requirements of conventional MRAC designs. One possible way to determine a passifying multiplier L is using the LDU factorization, as shown in [30]. To make this work more self-contained, this is discussed in Subsection 2.2.5.

1.4.4 WSPR based MIMO MRAC

Consider the control given by (1.39) and the error equations (1.40) and (1.36)in CT. Assume that $W_m(s)K_p$ is WSPR. Then, by definition, one has

$$A_c^T P + P A_c = -Q\,, \tag{1.67}$$

$$PBK_p = H_o^T W\,. \tag{1.68}$$

for some SPD matrix W. Now, as in [39], using the factorization $W = S^T S$, $S \in \mathbb{R}^{M \times M}$ non singular, it can be shown that the Lyapunov function (compare with the function utilized for the conventional adaptive law (1.43))

$$V = x_e^T P x_e + tr\left[S\tilde{\theta}^T \Gamma^{-1} \tilde{\theta} S^T\right], \tag{1.69}$$

where Γ is SPD. Let the adaptive law be chosen as

$$\dot{\theta}^T = -e\omega^T \Gamma\,. \tag{1.70}$$

Then, the derivative of V is given by

$$\dot{V} = -e^T Q e + 2e^T W \tilde{\theta}^T \omega - 2tr\left[Se\omega^T \tilde{\theta} S^T\right], \tag{1.71}$$

$$\dot{V} = -e^T Q e + 2e^T W \tilde{\theta}^T \omega - 2\omega^T \tilde{\theta} W e\,, \tag{1.72}$$

and, since the last two terms cancel, we have

$$\dot{V} = -e^T Q e\,, \tag{1.73}$$

implying that the adaptation is globally stable and the tracking error e tends to zero asymptotically. It is important to notice that the matrix W is not used in the adaptive law. It is only used to prove the stability.

1.5 L1 Adaptive Control

A recently proposed control architecture has attracted notable attention in the past few years. The so-called L1 Adaptive Control (L1-AC), firstly published in [33]-[57] and later in the book [58], according to the authors, provides fast adaptation with guaranteed transient properties using high adaptation gain. The controller basically consists of a modified Model Reference Adaptive Control (MRAC) using an input filtered control, a state prediction loop and high-gain adaptation law with parameter projection.

The L1-AC has been applied to aerospace systems. In [59], flight tests in an Unmanned Aerial Vehicle (UAV) showed robustness of the L1-AC. Robustness of the L1-AC was illustrate using the Rohrs counterexample as benchmark, where the conventional MRAC failed to maintain stability in the presence of unmodeled dynamics. Control of sattelites using the L1-AC is presented in [60], motivated by the use of high gain fast adaptation with quantifiable transient bounds. The interest in uniformly bounded transient lead to the the use a modified parametrization of L1-AC in [61], where the application is control of unmanned quadrotor aircrafts.

An overview of the application of L1-AC to aerospace systems, as well as a more extensive list of applications, is seen on [62]. Fundamental control characteristics of flying applications and discussion regarding what makes L1-AC suitable in this case is found in [63].

Other recent work can also be found on the literature reporting successful applications of the L1-AC. In [64] L1-AC is used to compensate hysteresis in piezoelectric actuators. Experimental results are presented showing good performance. In [65], the ability to provide guaranteed transient behavior motivated the extension of L1-AC to infinite-dimensional systems, where it is then applied to the design of adaptive observers and controllers.

The basic formulation of L1-AC is presented in this section. We revisit this technique in Chapter 4, where its drawbacks are discussed and a comparison with a newly presented controller is established.

1.5.1 L1-AC Problem Formulation

This subsection is based on [58].

Consider the class of systems

$$\dot{x}(t) = Ax(t) + b(u(t) + \theta^T x(t)), \quad x(0) = x_0 \tag{1.74}$$

$$y(t) = c^T x(t); \tag{1.75}$$

where $x(t) \in \mathbb{R}^n$ is the system state vector (measured); $u(t)$ is the control signal; $b, c \in \mathbb{R}^n$ are known constant vectors; $A \in \mathbb{R}^{n\times n}$ is a known $n \times n$ matrix with (A, b) controllable; θ is the unknown parameter, which belongs to a compact set and $y(t) \in \mathbb{R}$ is the regulated output.

The authors in [58] present an adaptive control solution such that $y(t)$ follows a given bounded and picewise-continuous reference signal $r(t)$ with quantfiable transient and steady-state performance bounds.

To that end, the following control structure is used

$$u(t) = u_m(t) + u_{ad}(t), \quad u_m(t) = -k_m^T x(t) \tag{1.76}$$

where k_m renders $A_m \triangleq A - bk_m^T$ Hurwitz, while $u_{ad}(t)$ is an adaptive component soon to be defined. The partially closed-loop system is given by:

$$\dot{x}(t) = A_m x(t) + b(\theta^T x(t) + u_{ad}(t)), \quad x(0) = x_0 \tag{1.77}$$

$$y = c^T x(t) \tag{1.78}$$

The following state-predictor is used

$$\dot{\hat{x}}(t) = A_m\hat{x}(t) + b(\hat{\theta}^T x(t) + u_{ad}(t)) \quad \hat{x}(0) = \hat{x}_0; \tag{1.79}$$
$$\hat{y} = c^T\hat{x}(t) \tag{1.80}$$

where $\hat{x}(t) \in \mathbb{R}^n$ is the state of the predictor and $\hat{\theta}(t) \in \mathbb{R}^n$ is the estimate of the parameter θ, obtained by a projection based adaptive law

$$\dot{\hat{\theta}} = \gamma Proj(\hat{\theta}(t), -\tilde{x}^T(t)Pbx(t)), \quad \hat{\theta}(0) = \hat{\theta}_0 \in \Theta; \tag{1.81}$$

the prediction error is defined as $\tilde{x}(t) \triangleq \hat{x}(t) - x(t)$, $\gamma \in \mathbb{R}^+$ is the adaptation gain, $P = P^T$ is the solution of Lyapunov equation $A_m^T P + PA_m = -Q$ for arbitrary symmetric $Q = Q^T > 0$. The projection is confined to the set Θ. The adaptive control signal in the frequency domain is

$$u_{ad}(s) = -C(s)(\hat{\eta}(s) - k_g r(s)) \tag{1.82}$$

where $r(s)$ and $\hat{\eta}(s)$ are the Laplace transforms of $r(t)$ and $\hat{\eta}(t) = \hat{\theta}^T(t)x(t)$, respectively. The input gain $k_g \triangleq -1/(c^T A_m^{-1} b)$ is assumed known and $C(s)$ is a stable filter. The L1-AC architecture is presented in the block diagram of Fig. 1.2.

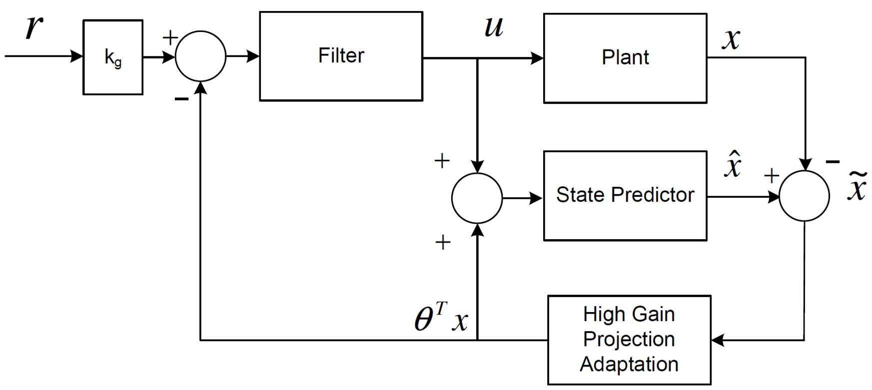

Figure 1.2: L1-AC architecture

The L1-AC is defined by Eq. (1.76),(1.79)-(1.82). A $\mathcal{L}_1$ condition on k_m and $C(s)$ is required as follows:

$$\lambda \triangleq \|G(s)\|_1 L < 1 \tag{1.83}$$

with

$$G(s) \triangleq H(s)(1 - C(s)), \quad H(s) \triangleq (sI - A_m)^{-1}b, \quad L \triangleq \max_{\theta\in\Theta} \|\theta\|_1 \tag{1.84}$$

The L1-AC considers a reference system defined by the nonadaptive version of the adaptive control system in Eq. (1.75), (1.76), (1.82) as follows

$$\dot{x}_{ref}(t) = Ax_{ref}(t) + b(\theta^T x_{ref}(t) + u_{ref}(t)), \quad x_{ref}(0) = x_0, \tag{1.85}$$

$$u_{ref}(s) = -C(s)(\theta^T x_{ref}(s) - k_g r(s)) - k_m^T x_{ref}(s), \tag{1.86}$$

$$y_{ref}(s) = c^T x_{ref}(s) \tag{1.87}$$

Note that the controller (1.86) attempts to compensate only for uncertainties within the bandwidth of $C(s)$.

Lemma 6 *If $\|G(s)\|_1 L < 1$, then the system (1.86) is bounded-input-bounded-state (BIBS) stable with respect to $r(t)$ and x_0*

(Proof: see [58]) ■

The following expression to the prediction error dynamics is obtained from Eqs. (1.77) and (1.79)

$$\dot{\tilde{x}}(t) = A_m\tilde{x}(t) + b\tilde{\theta}^T(t)x(t), \quad \tilde{x}(0) = 0 \tag{1.88}$$

with it is defined $\tilde{\theta}(t) = \hat{\theta}(t) - \theta$. Two lemmas are stated in [58] for the prediction error.

Lemma 7 *The prediction error in (1.88) is uniformly bounded:*

$$\|\tilde{x}\|_\infty \leq \sqrt{\frac{\theta_{max}}{\lambda_{min}(P)\Gamma}}, \quad \theta_{max} \triangleq 4\max_{\theta\in\Theta}\|\theta\|^2 \tag{1.89}$$

where $\lambda_{min}(P)$ is the minimum eigenvalue of P

Lemma 8 *If $u_{ad}(t)$ is defined according to (1.82) and the condition (1.83) holds, then we have the following asymptotic result:*

$$\lim_{t\to\infty}\tilde{x}(t) = 0$$

These two lemmas ensure convergence and guaranteed transient properties on the prediction error. We note, however, that at this point no guarantees are yet given to the tracking error.

The following theorem L1-AC is stated as found in [58] and defines the system state properties related to the reference system of Eq. (1.86).

Theorem 2 (L1-AC Theorem) *For the system in (1.75) and the controller defined via (1.76) and (1.79)-(1.82) subject to the $\mathcal{L}_1$-norm condition in (1.83), we* *have*

$$\|x_{ref} - x\|_\infty \leq \frac{\gamma_1}{\sqrt{\Gamma}}, \qquad \|u_{ref} - u\|_\infty \leq \frac{\gamma_2}{\sqrt{\Gamma}} \tag{1.90}$$

$$\lim_{t\to\infty} \|x_{ref} - x\| = 0, \qquad \lim_{t\to\infty} \|u_{ref} - u\| = 0, \tag{1.91}$$

where

$$\gamma_1 \triangleq \frac{\|C(s)\|_1}{1 - \|G(s)\|_1 L}\sqrt{\frac{\theta_{max}}{\lambda_{min}(P)}}, \tag{1.92}$$

$$\gamma_2 \triangleq \|H_1(s)\|_1 \sqrt{\frac{\theta_{max}}{\lambda_{min}(P)}} + \left\|C(s)\theta^T + k_m^T\right\|_1 \gamma_1 \tag{1.93}$$

(Proof: see [58]) ■

Theorem 2 implies that $x(t)$ and $u(t)$ can be made arbitrarily close to $x_{ref}(t)$ and $u_{ref}(t)$ by increasing the adaptive gain Γ. The control objective is then reduced to selecting k_m and $C(s)$ to obtain a desired response in the reference system.

For stabilization purposes, the following remark of [58] should be highlighted

Remark 1 *Since $C(0) = 1$, application of Final Value Theorem to the closed-loop reference system in (1.86) in the case of constant $r(t) \equiv r$ leads to*

$$\lim_{t\to\infty} y_{ref}(t) = c^T H(0)C(0)k_g r = r$$

Which along Theorem 2 indicates that a guaranteed performance can be obtained for constant reference signals.

It should be also noted that the L1-AC algorithm we presented here deals with SISO systems with known input gain and state feedback. Different variations of the L1-AC are found in the literature, so as it is possible to deal with unknown input gain, output feedback, nonlinear systems, MIMO systems, among other classes of plants. Nevertheless, it is interesting to note that these different applications often require significant changes in the controller architecture. For a more complete reference on L1-AC, see [58].

Despite reports of successful applications, some recent work question the efficiency of the L1-AC such as [66] and [67]. Criticisms include the use of excessively high adaptation gains and the inability to track a time-varying reference. The drawbacks of L1-AC are further discussed in Chapter 4.

1.6 Objective and Contribution of the Work

The objective of this Thesis is to propose new MRAC designs that does not require stringent symmetry assumptions on the HFG and to obtain robust controllers with guaranteed transient properties.

We seek to relax the stringent gain symmetry conditions on the plant HFG without the need of overparametrization and for plants of arbitrary relative degree. We thus propose techniques that require the PDJ condition on the HFG, which is a less restrictive condition.

In Chapter 2, we propose a new MRAC algorithm based on the bilinear error formulation, which is similar to conventional techniques. Circumvention of the symmetry assumption is achieved by means of a new adaptation law, such that the new controller is simple. Also, it can be applied to plants with arbitrary relative degree.

Since pure gradient adaptive control suffers from lack of robustness and may present poor adaptation transient, this has motivated the proposal of an extension to the BMRAC (Binary Model Reference Adaptive Control) [32]. The BMRAC consists basically of the conventional MRAC modified by parameter projection combined with high adaptation gain. and its MIMO extension was recently proposed in [30]. The BMRAC tends to behave as a sliding mode controller as the adaptation gain is increased. However, such gain can be tuned up to a sufficient value while avoiding chattering. Even though this solution also requires only that K_p is PDJ, thus relaxing the symmetry condition, it was restricted to plants of uniform relative degree one.

In Chapter 3, we propose a further extension to MIMO BMRAC to encompass non-uniform arbitrary relative degree plants that achieves global exact tracking. The relative degree obstacle is circumvented by using a hybrid estimation scheme recently generalized to a multivariable framework [68]. Such estimator, named, Global Robust Exact Differentiator (GRED), switches between a standard MIMO lead filter and a nonlinear one which utilizes Robust Exact Differentiators (RED) [69] based on higher order sliding modes. The use of MIMO GRED renders the error system uniformly globally exponentially practically stable with respect to a small residual set with ultimate convergence to zero.

The reason why we only dealt with a more elementary architecture of L1-AC in the previous section is because we are interested on its main features: the high gain adaptation with projection, the use of a prediction loop and the control input filtering. These ingredients are present in other previously proposed techniques as the Binary MRAC (BMRAC) [70] and the Smooth Sliding Control (SSC) [71]. These controllers present robustness and good transient behavior,

The Smooth Sliding Control (SSC), proposed by Hsu in [71] as a solution to avoid chattering in sliding mode control (SMC) systems, also relies on input filtered control together with an output error prediction loop, similarly to the L1-AC. The high-gain naturally appears since the control signal is generated by an amplitude modulated relay function. The discontinuous control is filtered prior to being injected into the plant, providing a smooth control signal. Despite the similarity, however, an essential difference is apparent since the SSC explicitly employs a reference model - which L1-AC does not. It is then possible to track a time-varying reference with a small residual error with the SSC while such property is not guaranteed with the L1-AC.

The use of high gain projection adaptation laws was proposed by [70], under the designation of Binary-MRAC (BMRAC), as a method to improve adaptation transient and to achieve the good performance and robustness properties of a sliding mode controller (SMC), while avoiding chattering. it can be shown that the BMRAC tends to a sliding mode controller as the adaptation gain was increased. Therefore, it can be expected that the SMC can be replaced by a BMRAC loop.

In Chapter 4, we propose a new controller obtained by a combination of BMRAC and SSC that shares the main features of L1-AC. The new controller is shown to present desirable robustness and guaranteed transient characteristics and is able to overcome certain limitations found in the L1-AC.

Also, it is interesting to compare the eBMRAC performance to the regular BMRAC. It is expected the eBMRAC to inherit robustness properties from the SSC architecture, such that it would be an advantage in comparison to the BMRAC. This controller is presented in its SISO version with preliminar results to MIMO extension.

Chapter 2

Bilinear Error Multivariable MRAC

2.1 Introduction

Inspired by recent results related to the WSPR concept, we revisit the "conventional" MIMO direct MRAC design [2, 21] and propose a new design. The significant advantage is that, while keeping similar structure and complexity as in the conventional design, the stability can be established without requiring the aforementioned symmetry conditions.

This chapter proposes a new multivariable MRAC design for plants with arbitrary (vector) relative degree, based on the PDJ condition. Both Continuous-Time and Discrete-Time cases are considered.

In order to extend the MIMO MRAC to plants which do not satisfy the PDJ condition, we proceed as explained in [44]. We choose an adequate matrix multiplier L such that LK_p does satisfy such condition in a robust way, i.e, the condition holds for an open set in the K_p space. Furthermore, we propose a modified adaptation law using right gain multipliers so that global stability and error convergence to zero can be proved for the MIMO MRAC. We show that, under certain circumstances, the symmetry requirements can be relaxed in the conventional design.

To this end, we first establish a fundamental lemma about the boundedness of the normalized estimation error, the estimated parameters, and their derivatives or variations. Such lemma leads to the complete stability and convergence proof of the closed-loop adaptive system, following a well established stability analysis [21, pp.405].

2.2 Bilinear MIMO MRAC Design using a stabilizing multiplier

Our new controller inherits the design approach based on a bilinear error formulation as in the conventional MIMO MRAC and its basic structure. Since the control signal is obtained through different update laws, we present the problem formulation as in the conventional case and thus propose a new update law. We use “conventional” to refer to MIMO MRAC techniques found in textbooks such as [2] and [21]. In this way, a direct comparison between both approaches is easily established, showing that the new controller is similar to the conventional one. However, with a important difference in the choice of new adaptation laws.

2.2.1 Error Equations

The output error and input signal are given by (1.25) and (1.31) and repeated here for convenience

$$e = y - y_m; \quad u = \theta^T \omega \tag{2.1}$$

The control objective is to find a feedback control $u(t)$ such that the output error tends asymptotically to zero for arbitrary initial conditions and uniformly bounded picewise continuous reference signals $r(t)$.

For both the continuous and discrete-time cases, the error is thus obtained from (1.31), (1.25) and (1.37).

$$(y - y_m) = W_m(D) K_p \left[(\theta - \theta^*)^T \omega\right] \tag{2.2}$$

Without loss of generality, choose $W_m(D) = \xi_m(D)^{-1}$ from Eq. (1.23):

$$\xi_m(D)(y - y_m) = K_p \left[(\theta - \theta^*)^T \omega\right] \tag{2.3}$$

Let δ_m denote the maximum degree of $\xi_m(D)$. Then, filtering both sides by $h(D) = \frac{1}{f(D)}$ where $f(D)$ is any stable polynomial (Hurwitz, in CT, or Schur, in DT) of degree δ_m, we get

$$\xi_m(D) h(D)(y - y_m) = K_p \left[h(D)[u] - {\theta^*}^T h(D)[\omega]\right] \tag{2.4}$$

And then define the normalized error, with ψ being the estimate of $\psi^* = K_p$

$$\epsilon = \frac{\xi_m(D) h(D)[y - y_m] + \psi \xi}{1 + \beta(\zeta^T \zeta + \xi^T \xi)} \tag{2.5}$$

with the signals $\xi(t)$ and $\zeta(t)$ defined as

$$\xi(t) = \theta^T(t)\zeta(t) - h(D)[\theta^T\omega](t) \tag{2.6}$$

$$\zeta(t) = h(D)[\omega](t) \tag{2.7}$$

From (2.4), (2.5) and (2.6), defining $\tilde{\psi} = \psi - \psi^*$ and $\tilde{\theta} = \theta - \theta^*$ and noting that $u = \theta^T\omega$

$$\epsilon = \frac{K_p\left[h(D)[u](t) + \tilde{\theta}^T h(D)[\omega](t) - \theta^T h(D)[\omega](t)\right]}{1+\beta(\zeta^T\zeta + \xi^T\xi)} + \\ + \frac{\psi(t)\theta^T(t)\zeta(t) - \psi(t)h(D)[u](t)}{1+\beta(\zeta^T\zeta+\xi^T\xi)} \tag{2.8}$$

Finally, since $\psi^* = K_p$ and from (2.6) and (2.7), we find the bilinear parametric error model [2].

$$\epsilon = \frac{\tilde{\psi}\xi + K_p\tilde{\theta}^T\zeta}{m^2} \tag{2.9}$$

where m^2 is a normalizing signal defined as follows with $\beta > 0$.

$$m^2 = 1 + \beta(\zeta^T\zeta + \xi^T\xi) \tag{2.10}$$

It is easy to conclude that such normalization implies that the estimation error ϵ is bounded if the parameters are bounded, irrespectively of the signals ξ and ζ provided they are L_e^∞.

2.2.2 Conventional Design

Using the previous error model, the conventional design [21] requires the knowledge of an $M \times M$ matrix S_p such that $K_pS_P = (K_pS_p)^T > 0$, for the CT case, or $2I > K_pS_P = (K_pS_p)^T > 0$, in the DT case. Then, the CT parameter update laws are given by

$$\dot{\theta}^T(t) = -S_p\epsilon(t)\zeta^T(t) \tag{2.11}$$

$$\dot{\psi}(t) = -\Gamma\epsilon(t)\xi^T(t) \tag{2.12}$$

For the DT case we have:

$$\theta^T(t+1) = \theta^T(t) - S_p\epsilon(t)\zeta^T(t) \tag{2.13}$$

$$\psi(t+1) = \psi(t) - \Gamma\epsilon(t)\xi^T(t) \tag{2.14}$$

where Γ is an $M \times M$ constant adaptation gain matrix such that $\Gamma = \Gamma^T > 0$ ($2\beta I > \Gamma$ for the DT case).

It is particularly important to note here that, in this scheme, the matrix S_p has to be known and is used in the adaption law. This is a specially restrictive condition if K_p is uncertain since symmetry is a nongeneric property and could be destroyed with an arbitrarily small modification of K_p, within the uncertainty domain.

Stability can be proven using the error model of Eq. (2.9) and the following Lyapunov-like function:

$$V(\tilde{\theta}(t), \tilde{\psi}(t)) = \frac{1}{2}tr[\tilde{\theta}(t)\Gamma_p^{-1}\tilde{\theta}^T(t)] + \frac{1}{2}tr[\tilde{\psi}^T(t)\Gamma^{-1}\tilde{\psi}(t)] \tag{2.15}$$

with $\Gamma_p = S_p K_p^{-T}$, and $t \in [0, \infty)$ for the CT case and $t \in \{0, 1, 2, 3, ...\}$ for the DT case.

2.2.3 New Design

The new solution uses the following adaptation laws for the CT case. The multiplier $\bar{L}$ is such that $\bar{L}K_p$ is PDJ and is used to ensure stability properties for the closed loop system.

$$\dot{\theta}^T(t) = -\bar{L}\epsilon(t)\zeta^T(t)\Gamma_1 \tag{2.16}$$
$$\dot{\psi}(t) = -\epsilon(t)\xi^T(t)\Gamma_2 \tag{2.17}$$

and, for the DT case

$$\theta^T(t+1) = \theta^T(t) - \bar{L}\epsilon(t)\zeta^T(t)\Gamma_1 \tag{2.18}$$
$$\psi(t+1) = \psi(t) - \epsilon(t)\xi^T(t)\Gamma_2 \tag{2.19}$$

where Γ_1 and Γ_2 are arbitrary symmetric positive definite matrices. Note here that the gain matrices Γ_1, Γ_2 are right multipliers (they weight the elements of the regressors vectors ζ, ξ), while in the conventional laws, are left multipliers (they weight the elements of the error vector ϵ). Moreover, note that in the new design, the gains Γ_1, Γ_2 are arbitrary, while in the conventional case the gain S_p explicitly involves the symmetrization of $S_p K_p$.

2.2.4 Stability Analysis

We wish to establish the following fundamental lemma.

Lemma 9 *Supposing that a matrix $\bar{L}$ is known such that $\bar{L}K_p$ is PDJ, the following stability properties can be established:*

(i) The adaptive law (2.16), (2.17) ensures that $\theta(t)$, $\psi(t)$, $\epsilon(t) \in L^\infty$, $\dot{\theta}(t)$, $\dot{\psi}(t)$, $\epsilon(t)m(t) \in L^\infty$, *and*

$$\int_{t_1}^{t_2} x^2(t)dt \le a_c \tag{2.20}$$

for some constant $a_c > 0$*, any* $t_1, t_2 \ge 0$*,* $x(t) = \|\dot{\theta}(t)\| + \|\epsilon(t)\|m(t)$

(ii) The adaptive law (2.18), (2.19) ensures that $\theta(t)$, $\psi(t)$, $\epsilon(t), e(t)m(t) \in L^\infty$, *and*

$$\sum_{t=t_1}^{t_2-1} x^2(t) \le a_d \tag{2.21}$$

for some constant $a_d > 0$*, any* $t_1, t_2 \ge 0$*,* $x(t) = \|\theta(t+1) - \theta(t)\| + \|\epsilon(t)\|m(t)$*.*

■

If this lemma holds, the closed loop stability and tracking error asymptotic convergence can be proved following a well established analysis as found in the textbook by Tao [21, pp. 405].

Continuous-time

The proof of Lemma 9 for the CT case follows.

Proof: As in [39], we consider the following factorizations to propose a Lyapunov-like function candidate, $W_1 = S_1^T S_1, S_1 \in \mathbb{R}^{M\times M}$ nonsingular, $W_2 = S_2^T S_2, S_2 \in \mathbb{R}^{M\times M}$ nonsingular.

$$V(\tilde{\theta}, \tilde{\psi}) = \frac{1}{2} tr[S_1 \tilde{\theta}^T \Gamma_1^{-1} \tilde{\theta} S_1^T] + \frac{1}{2} tr[S_2 \bar{L} \tilde{\psi} \Gamma_2^{-1} \tilde{\psi}^T \bar{L}^T S_2^T] \tag{2.22}$$

whose time derivative is:

$$\dot{V} = tr[S_1 \dot{\tilde{\theta}}^T \Gamma_1^{-1} \tilde{\theta} S_1^T] + tr[S_2 \bar{L} \dot{\tilde{\psi}} \Gamma_2^{-1} \tilde{\psi}^T \bar{L}^T S_2^T] \tag{2.23}$$

Substituting (2.16) and (2.17) in (2.23), we have

$$\dot{V} = tr[-S_1 \bar{L} \epsilon \zeta^T \tilde{\theta} S_1^T] + tr[-S_2 \bar{L} \epsilon \xi^T \tilde{\psi}^T \bar{L}^T S_2^T] \tag{2.24}$$

Since $tr(xy^T) = y^T x$, it is possible to rewrite (2.24) as $\dot{V} = -(\zeta^T \tilde{\theta} W_1 \bar{L} \epsilon + \xi^T \tilde{\psi}^T \bar{L}^T W_2 \bar{L} \epsilon)$ with $W_1 = S_1^T S_1$ and $W_2 = S_2^T S_2$ being SPD matrices. This can be rewritten as

$$\dot{V} = -(\zeta^T \tilde{\theta} W_1 + \xi^T \tilde{\psi}^T \bar{L}^T W_2) \bar{L} \epsilon \;=\; -\epsilon^T \bar{L}^T (W_1 \tilde{\theta}^T \zeta + W_2 \bar{L} \tilde{\psi} \xi) \tag{2.25}$$

Defining $W_1 = W_2 \bar{L} K_p$ such that $W_2^{-1} W_1 = \bar{L} K_p$, one has $\dot{V} = -\epsilon^T \bar{L}^T W_2 \bar{L}(K_p \tilde{\theta}^T \zeta +$

$\tilde{\psi}\xi$). Thus, it leads to

$$\dot{V} = -m^2[\epsilon^T \bar{L}^T]W_2\bar{L}\epsilon = -m^2[\epsilon^T \bar{L}^T W_2 \bar{L}\epsilon] \leq 0 \tag{2.26}$$

Note that $\bar{L}K_p$ must be anti-Hurwitz, since $W_1 = W_2\bar{L}K_p$, can be written as $K_p^T\bar{L}^T W_2 + W_2\bar{L}K_p = 2W_1$ since W_1 and W_2 are symmetric. We should emphasize that, even though symmetry is required for W_1 and W_2, these matrices are just for the stability analysis and do not need to be known i.e. only their existence must be guaranteed. This holds if $\bar{L}K_p$ is PDJ.

It is interesting to observe that de PDJ condition appears as a technicality in the stability proof and it is not associated to any property of the system. In the generalized passivity framework, where this condition was originally established, the PDJ condition is associated to a passivity property.

This result guarantees that $\theta(t), \psi(t) \in L^\infty$. Then, from 2.9, one concludes that $\epsilon(t) \in L^\infty$. Indeed, note that in (2.9) and (2.10) the denominator contains the squared norms ζ and ξ while these signals appear linearly in the numerator. From the update laws (2.16) and (2.17), it follows that $\dot{\theta}(t), \dot{\psi}(t), \epsilon(t)m(t) \in L^\infty$ due to the normalization (ζ and ξ appear quadratically in the numerator and also in the denominator). Integrating both sides of (2.26) and from the uniform boundedness of $V(t)$, one concludes that $\|\epsilon m\| \in L^2$. Noting that (2.16) can be rewritten as $\dot{\tilde{\theta}}^T = -m\epsilon(\zeta^T/m)\Gamma_1$ and since $\zeta^T/m \in L^\infty$ then $\|\dot{\tilde{\theta}}\| \in L^2$. ■

The necessary condition for the existence of a SPD W_2 such that $W_1 = W_2\bar{L}K_p$ is also SPD is that $\bar{L}K_p$ must have real positive eigenvalues and diagonal Jordan form (PDJ) [44, Lemma 3]. This is indeed less restrictive than the symmetry requirement and, moreover, it is a non fragile and generic condition.

Discrete-time

Proof: We use the same Lyapunov-like candidate considered in the previous case

$$V(\tilde{\theta}(t), \tilde{\psi}(t)) = \frac{1}{2}tr[S_1\tilde{\theta}^T\Gamma_1^{-1}\tilde{\theta}S_1^T] + \frac{1}{2}tr[S_2\bar{L}\tilde{\psi}\Gamma_2^{-1}\tilde{\psi}^T\bar{L}^T S_2^T] \tag{2.27}$$

From (2.18) and (2.19), it follows that

$$\begin{aligned} \Delta V = V(\tilde{\theta}(t+1), \tilde{\psi}(t+1)) - V(\tilde{\theta}(t), \tilde{\psi}(t)) = \\ = -tr[S_1\bar{L}\epsilon(t)\zeta^T(t)\tilde{\theta}(t)S_1^T] - tr[S_2\bar{L}\epsilon(t)\xi^T(t)\tilde{\psi}^T(t)\bar{L}^T S_2^T] + \\ +\frac{1}{2}tr[S_1\bar{L}\epsilon(t)\zeta^T(t)\Gamma_1\zeta(t)\epsilon^T(t)\bar{L}^T S_1^T] + \\ +\frac{1}{2}tr[S_2\bar{L}\epsilon(t)\xi^T(t)\Gamma_2\xi(t)\epsilon^T(t)\bar{L}^T S_2^T] \end{aligned} \tag{2.28}$$

Using the property $tr(xy^T) = y^T x$, it follows that

$$\begin{aligned} \Delta V &= -\zeta^T\tilde{\theta}W_1\bar{L}\epsilon + \frac{1}{2}(\zeta^T\Gamma_1\zeta(\bar{L}\epsilon)^T W_1\bar{L}\epsilon) + \\ &- (\xi^T\tilde{\psi}^T\bar{L}^T W_2)\bar{L}\epsilon + \frac{1}{2}((\bar{L}\epsilon)^T W_2\bar{L}\epsilon\xi^T\Gamma_2\xi) \end{aligned} \tag{2.29}$$

where, as in the previous case, $W_1 = S_1^T S_1$ and $W_2 = S_2^T S_2$ are SPD matrices. Defining $W_1 = W_2\bar{L}K_p$ and noting that $W_1 = W_2\bar{L}K_p = K_p^T\bar{L}^T W_2 = W_1^T$, and from (2.9), one has that

$$\begin{aligned} \Delta V &= -(\zeta^T\tilde{\theta}K_p^T\bar{L}^T - \xi^T\tilde{\psi}^T\bar{L}^T)W_2\bar{L}\epsilon + \\ &+ \frac{1}{2}((\bar{L}\epsilon)^T W_1\bar{L}\epsilon)(\zeta^T\Gamma_1\zeta) + \frac{1}{2}((\bar{L}\epsilon)^T W_2\bar{L}\epsilon)(\xi^T\Gamma_2\xi) \end{aligned} \tag{2.30}$$

Note that $\zeta^T\tilde{\theta}K_p^T - \xi^T\tilde{\psi}^T = \epsilon^T m^2$, and since $m^2 = 1 + \beta(\zeta^T\zeta + \xi^T\xi)$, this leads to

$$\begin{aligned} \Delta V &= -[1 + \beta(\zeta^T\zeta + \xi^T\xi)]((\bar{L}\epsilon)^T W_2\bar{L}\epsilon) + \\ &+ \frac{1}{2}((\bar{L}\epsilon)^T W_1\bar{L}\epsilon)(\zeta^T\Gamma_1\zeta) + \frac{1}{2}((\bar{L}\epsilon)^T W_2\bar{L}\epsilon)(\xi^T\Gamma_2\xi) \end{aligned} \tag{2.31}$$

Thus, the following inequality can be derived

$$\begin{aligned} \Delta V \leq -(\bar{L}\epsilon)^T W_2\bar{L}\epsilon - \beta(\zeta^T\zeta)((\bar{L}\epsilon)W_2\bar{L}\epsilon) - \beta(\xi^T\xi)((\bar{L}\epsilon)^T W_2\bar{L}\epsilon) + \\ + \frac{1}{2}\lambda_{max}(\Gamma_1)\zeta^T\zeta((\bar{L}\epsilon)^T W_1\bar{L}\epsilon) + \frac{1}{2}\lambda_{max}(\Gamma_2)\xi^T\xi((\bar{L}\epsilon)^T W_2\bar{L}\epsilon) \end{aligned}$$

which can be further developed to

$$\begin{aligned} \Delta V \leq -(\bar{L}\epsilon)^T W_2\bar{L}\epsilon - \zeta^T\zeta\left[\epsilon^T\bar{L}^T\left(\beta W_2 - \frac{1}{2}\ \lambda_{max}(\Gamma_1)W_2\bar{L}K_p\right)\bar{L}\epsilon\right] + \\ -\xi^T\xi\left[\beta - \frac{1}{2}\lambda_{max}(\Gamma_2)\right]\left[(\bar{L}\epsilon)^T W_2\bar{L}\epsilon\right] \end{aligned} \tag{2.32}$$

Thus, to ensure that $\Delta V \leq 0$, β is chosen to satisfy the inequalities

$$\beta \geq \frac{1}{2}\lambda_{max}(\Gamma_2) \tag{2.33}$$

$$\beta W_2 - \frac{1}{2}\lambda_{max}(\Gamma_1)W_2\bar{L}K_p \geq 0 \tag{2.34}$$

It is important to note that W_2 is only used for analysis purposes, which makes it desirable to obtain a condition on β that does not depend on W_2. A simpler expression is derived as follows.

Since W_2 e $W_2\bar{L}K_p$ are symmetric, then $(\beta W_2 - \frac{1}{2}\lambda_{max}(\Gamma_1)W_2\bar{L}K_p)$ is also symmetric. Hence,

$$W_2\left(\beta I - \frac{1}{2}\lambda_{max}(\Gamma_1)\bar{L}K_p\right) = \left(\beta I - \frac{1}{2}\lambda_{max}(\Gamma_1)K_p^T\bar{L}^T\right)W_2 \tag{2.35}$$

Thus, it follows that

$$W_2(\beta I - \frac{1}{2}\lambda_{max}(\Gamma_1)\bar{L}K_p) = \frac{W_2}{2}(\beta I - \frac{1}{2}\lambda_{max}(\Gamma_1)\bar{L}K_p) + (\beta I - \frac{1}{2}\lambda_{max}(\Gamma_1)K_p^T\bar{L}^T)\frac{W_2}{2} \tag{2.36}$$

The above equation has the form $A^TW_2 + W_2A = Q$ (with $A = \beta I - \frac{1}{2}\lambda_{max}(\Gamma_1)\bar{L}K_p$). Thus, if $\lambda_i(A) > 0$, then $Q = Q^T > 0$. Therefore, (2.34) holds if

$$\beta > \frac{\lambda_{max}(\Gamma_1)\lambda_{max}(\bar{L}K_p)}{2} \tag{2.37}$$

Consequently, if the explicit inequality

$$\beta > \max\left(\frac{1}{2}\lambda_{max}(\Gamma_2), \frac{\lambda_{max}(\Gamma_1)\lambda_{max}(\bar{L}K_p)}{2}\right) \tag{2.38}$$

is satisfied then $\Delta V \leq -(\bar{L}\epsilon)^TW_2\bar{L}\epsilon$, as desired for stability. Finally, using similar arguments as for the CT case, the DT part of Lemma 9 also holds. ■

Table 2.1: Comparison between conventional and new design

Plant model	$y(t) = G(D)[u](t)$	
Reference Model	$y_m = W_m(D)$	
Error model	$\epsilon = \frac{\tilde{\psi}\xi + K_p\tilde{\theta}^T\zeta}{m^2}$	
Regressor Matrix	$\omega = [\omega_u^T\ \omega_y^T\ y^T\ r^T]^T$	
Filters	$\omega_u = A(D)\Lambda^{-1}(D)u$ $\omega_y = A(D)\Lambda^{-1}(D)y$	
Normalizing Signals	$\xi(t) = \theta^T(t)\zeta(t) - h(D)[\theta^T\omega](t)$ $\zeta(t) = h(D)[\omega](t)$ $m^2 = 1 + \beta(\zeta^T\zeta + \xi^T\xi)$ β is arbitrary positive in CT and has to satisfy (2.38) in DT	
Control Law	$u = \theta^T\omega$	
	Conventional Design	New Design
Adaptation Law	$CT: \begin{cases} \dot{\theta}^T(t) = -S_p\epsilon(t)\zeta^T(t) \\ \dot{\psi}(t) = -\Gamma\epsilon(t)\xi^T(t) \end{cases}$ $DT: \begin{cases} \theta^T(t+1) = \theta^T(t)+ \\ \quad - S_p\epsilon(t)\zeta^T(t) \\ \psi(t+1) = \psi(t)+ \\ \quad - \Gamma\epsilon(t)\xi^T(t) \end{cases}$	$CT: \begin{cases} \dot{\theta}^T(t) = -\bar{L}\epsilon(t)\zeta^T(t)\Gamma_1 \\ \dot{\psi} = -\epsilon(t)\xi^T(t)\Gamma_2 \end{cases}$ $DT: \begin{cases} \theta^T(t+1) = \theta^T(t)+ \\ \quad - \bar{L}\epsilon(t)\zeta(t)^T\Gamma_1 \\ \psi(t+1) = \psi(t)+ \\ \quad - \epsilon(t)\xi(t)^T\Gamma_2 \end{cases}$
Lyapunov Function	$V = tr[\tilde{\theta}\Gamma_p^{-1}\tilde{\theta}^T] + tr[\tilde{\psi}^T\Gamma^{-1}\tilde{\psi}]$	$V(\tilde{\theta},\tilde{\psi}) = \frac{1}{2}tr[S_1\tilde{\theta}^T\Gamma_1^{-1}\tilde{\theta}S_1^T]+$ $+\frac{1}{2}tr[S_2\bar{L}\tilde{\psi}\Gamma_2^{-1}\tilde{\psi}^T\bar{L}^TS_2^T]$

Remark 2 (Extension to the conventional design) *The result of Lemma 1*

can be applied to the conventional design assuming that $\Gamma = \gamma I$ and S_p is such that S_pK_p is PDJ. In this case, S_p plays the role of the stabilizing multiplier $\bar{L}$.

2.2.5 Determining the stabilizing multiplier $\bar{L}$

Assuming that a nominal value of K_p is known and that all leading minors are nonzero, then the following factorization always exists

$$K_p = L_pD_pU_p \tag{2.39}$$

where L_p is unit lower triangular, D_p is diagonal and U_p is unit upper triangular. Choosing a diagonal matrix D_0 with positive and distinct diagonal elements and with the matrices L_p and D_p of the LDU factorization of the nominal value of K_p, a lower triangular multiplier matrix $\bar{L}$ can be obtained

$$\bar{L} = D_0(L_pD_p)^{-1} \tag{2.40}$$

such that the matrix

$$\bar{K_p} = LK_p = D_0(L_pD_p)^{-1}(L_pD_p)U_p = D_0U_p \tag{2.41}$$

is upper triangular with diagonal elements and eigenvalues positive real and distinct. Thus, from [44, Lemma 3] there exists a matrix W such that WLK_p is SPD.

2.3 Simulation Results

In this section, we present some simulation results to illustrate the theoretical developments.

2.3.1 Process control plant with nonuniform relative degree satisfying the PDJ condition

The first case we address is inspired on a stable chemical process model presented in [72, pp.701], which is made unstable in order to address a more challenging control problem. We consider an uncertain MIMO LTI plant with nonuniform vector relative degree ($\rho_1 = 2, \rho_2 = 1$).

$$G(s) = \begin{bmatrix} \frac{4}{s^2+4s+3} & \frac{-2}{s^2+4s+3} \\ \frac{-1}{s-1} & \frac{1}{s-1} \end{bmatrix} \tag{2.42}$$

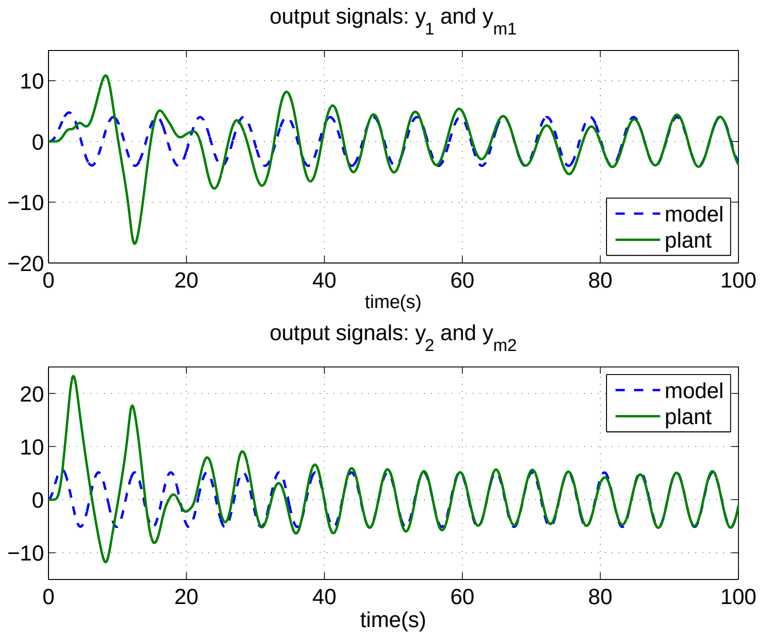

Figure 2.1: Process control plant tracking performance: plant output y (—); and model output y_m (- -)

Note that $K_p = lim_{s\to\infty}\xi(s)G(s)$ is PDJ but not symmetric

$$K_p = \begin{bmatrix} 4 & -2 \\ -1 & 1 \end{bmatrix} \tag{2.43}$$

We use the reference model:

$$W_m(s) = \begin{bmatrix} \frac{1}{s^2+2s+1} & 0 \\ 0 & \frac{1}{s+2} \end{bmatrix} \tag{2.44}$$

with reference signals $r_1 = 8\sin(t)$ and $r_2 = 8\sin(1.2t)$. We choose $\beta = 0.1$; the matrix gains are $\Gamma_1 = 0.25\ diag\{0.5, 1, 5, 1, 0.5, 1, 1, 1\}$, $\Gamma_2 = 0.25 I_{2\times 2}$; I/O state filters (1.29): $\Lambda(s) = \frac{1}{s+10}$; (observability index) $\nu = 2$; $h(s) = \frac{1}{(s+2)^2}$. The controller is designed assuming that there is no prior knowledge on the plant parameters, so that the estimated parameters initial conditions are set to zero. The remaining system initial conditions are also set to zero.

The tracking performance of the proposed controller can be seen in Fig. 2.1, and, as expected, the tracking errors converge to zero as shown in Fig. 2.2. As can be observed in Fig. 2.3, the control signal $u(t)$ converges to u^*.

2.3.2 Visual Servoing Application

Consider the problem of direct adaptive visual tracking for planar manipulators using a fixed camera (plant) with optical axis orthogonal to the robot workspace.

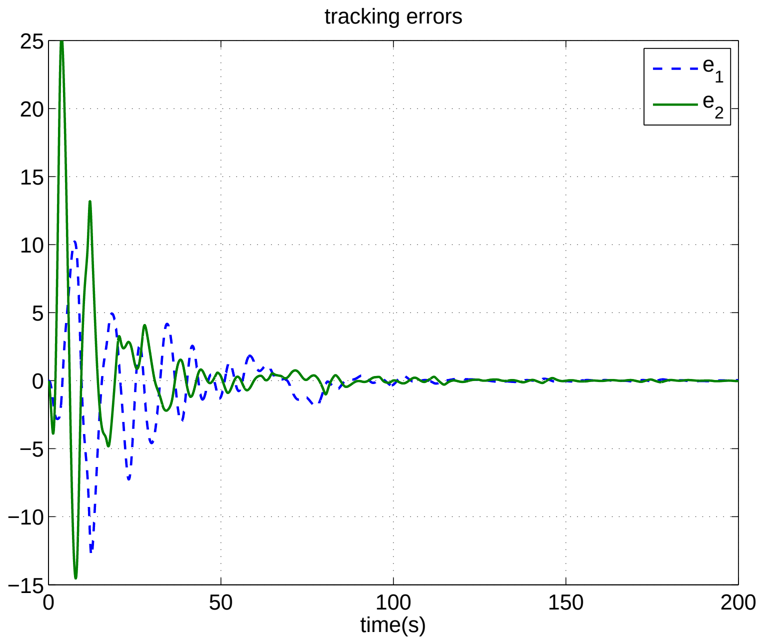

Figure 2.2: Process control plant: tracking errors

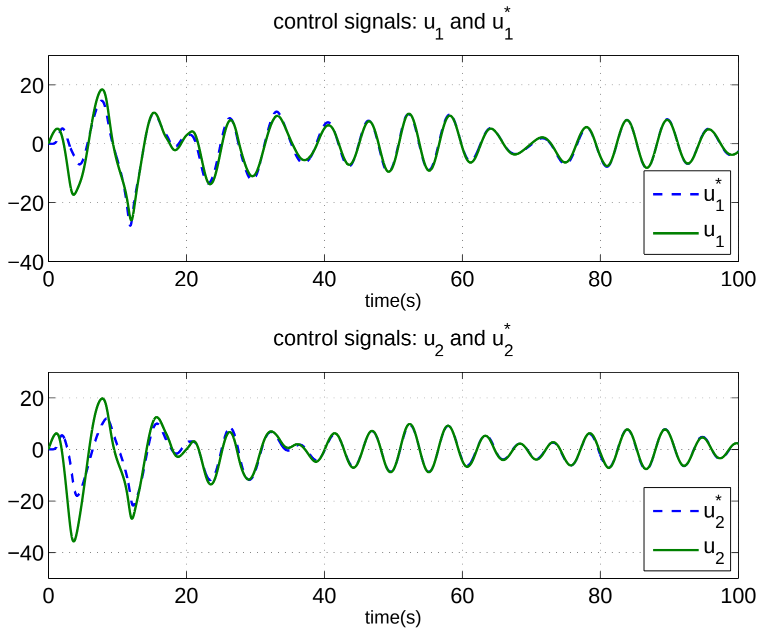

Figure 2.3: Process control plant: control signal u (—); and model matching signal u^* (- -)

The camera orientation is uncertain with respect to the robot workspace coordinates [44, 45]. The objective is to control the robot so that the image of its end-effector tracks a desired trajectory in the image plane. The objective is to control the robot

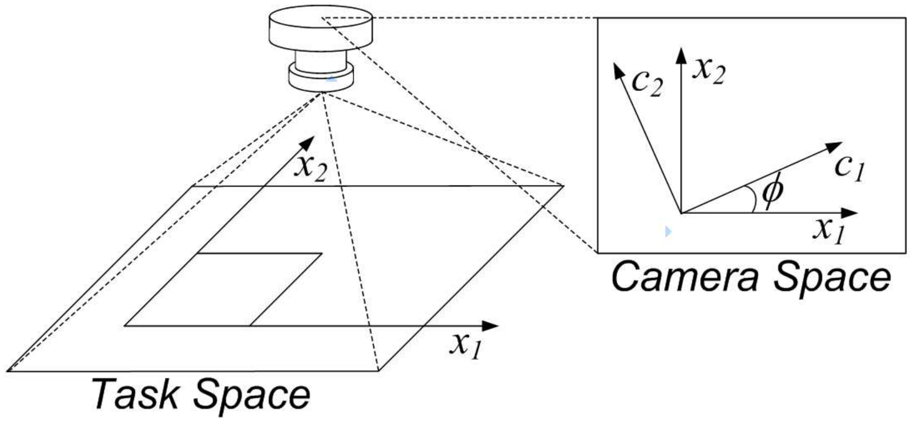

Figure 2.4: Camera-robot system representation

such that the image of the efectuator tracks the desired trajectory in the image plane.

The motivation to choose this example is that the HFG is essentially a rotation matrix which, except for the trivial cases, is neither non-symmetric nor PDJ since its eigenvalues are complex.

The associated cartesian control problem of the camera coordinate frame is described by:

$$\dot{x}_c = K_p u; \quad K_p = \begin{bmatrix} cos(\phi) & -sin(\phi) \\ sin(\phi) & cos(\phi) \end{bmatrix} \tag{2.45}$$

Where $x_c \in \mathbb{R}^2$ represents the coordinates of the efectuator in the image plane, $u \in \mathbb{R}^2$ is the cartesian control law. The HFG is the rotation matrix that represents the relationship between the image space and the robot workspace. Trajectory in the image plane is generated by the following reference model:

$$W_m(s) = \begin{bmatrix} \frac{1}{s+1} & 0 \\ 0 & \frac{1}{s+1} \end{bmatrix} \tag{2.46}$$

Considering the LDU decomposition of the nominal value of K_p

$$K_p = \begin{bmatrix} c & -s \\ s & c \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ s/c & 1 \end{bmatrix} \begin{bmatrix} c & 0 \\ 0 & 1/c \end{bmatrix} \begin{bmatrix} 1 & -s/c \\ 0 & 1 \end{bmatrix}, \tag{2.47}$$

the stabilizing multiplier $\bar{L}$ can be obtained as previously discussed, leading to

$$\bar{L} = D_0 \left[\begin{bmatrix} 1 & 0 \\ s/c & 1 \end{bmatrix} \begin{bmatrix} c & 0 \\ 0 & 1/c \end{bmatrix} \right]^{-1} = \begin{bmatrix} \kappa_1/c & 0 \\ -\kappa_1 s & \kappa_2 c \end{bmatrix}. \tag{2.48}$$

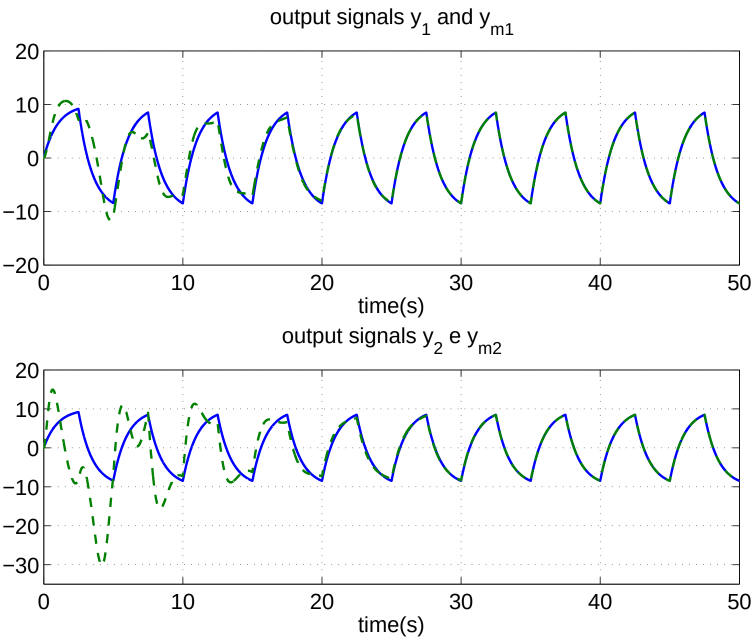

Figure 2.5: Visual servoing plant tracking performance: plant output y (—); and model output y_m (- -) with stabilizing multiplier

where $c = \cos(\phi)$ and $s = \sin(\phi)$. Defining $D_0 = diag\{\kappa_1, \kappa_2\}$ and replacing in $\bar{L} = D_0(L_p D_p)^{-1}$, we can evaluate $\bar{L}$ for some nominal value of ϕ fixing $\kappa_2 = 1$ and letting κ_1 be a free parameter. For a given nominal value of ϕ, denoted ϕ_{nom}, and a given κ_1, the range of camera orientation that allows the PDJ condition to hold for LK_p can be determined numerically as presented in [44]. The reference signals are chosen as $r_1 = r_2 = 20sqw(0.2t)$, where $sqw(t)$ denotes the unit amplitude square wave. The matrix gains are $\Gamma_1 = \Gamma_2 = 10I_{2\times 2}$ and we choose $\beta = 1$. Other design parameters are: I/O state filters (1.29): $\Lambda(s) = \frac{10}{s+3}$ and (observability index) $\nu = 1$; $h(s) = \frac{1}{(s+1)}$. The plant initial conditions are zero. The controller is designed assuming the nominal $\phi_{nom} = 45°$. The multiplier $\bar{L}$ is calculated according to Eq. (2.48) using $\kappa_1 = 50$ and $\kappa_2 = 1$. The real angle is set to $\phi = 75°$. The remaining system initial conditions are also set to zero.

The system tracking performance is significantly improved when the multiplier is used (see Fig. 2.7–2.6). With the stabilizing multiplier, the tracking errors converge to zero around $t = 50s$ as can be seen in Fig. 2.5 and Fig. 2.6 whilst in the other case convergence is not observed even for longer simulations (t=5000s).

2.3.3 Discrete-Time Plant

We consider a generic discrete-time second order plant with nonuniform relative degree. For the sake of simplicity, we firstly consider a plant with the same K_p as the process control plant, which is nonsymmetric and PDJ. Note that this plant is

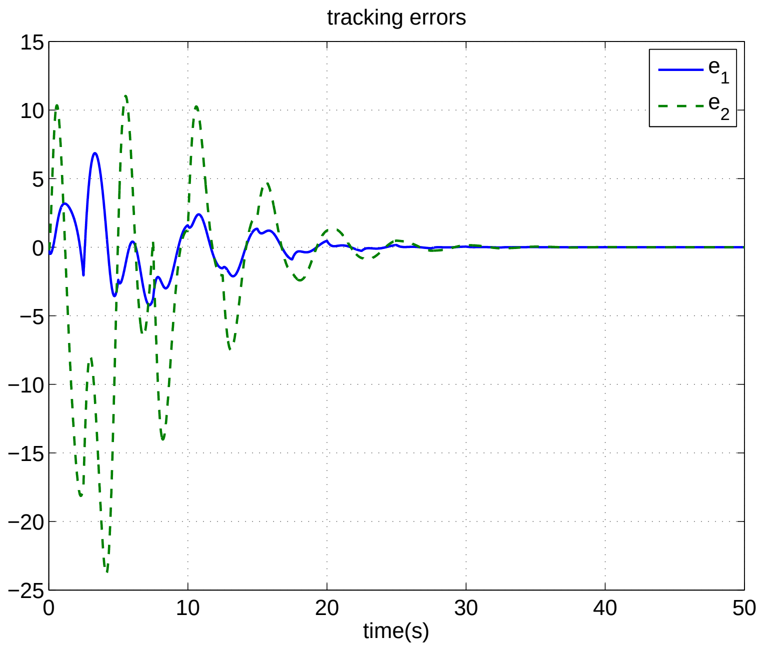

Figure 2.6: Visual servoing plant with the stabilizing multiplier: tracking errors

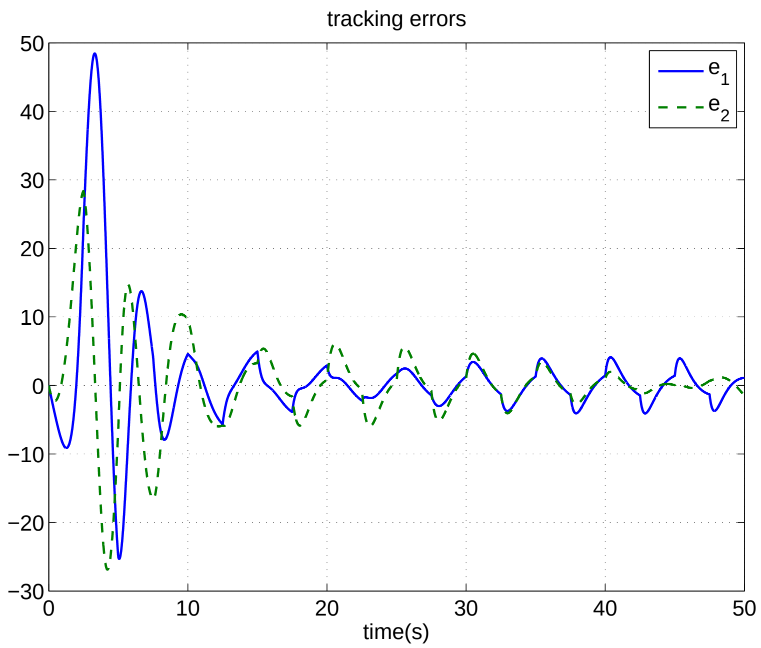

Figure 2.7: Visual servoing plant without the stabilizing multiplier: tracking errors

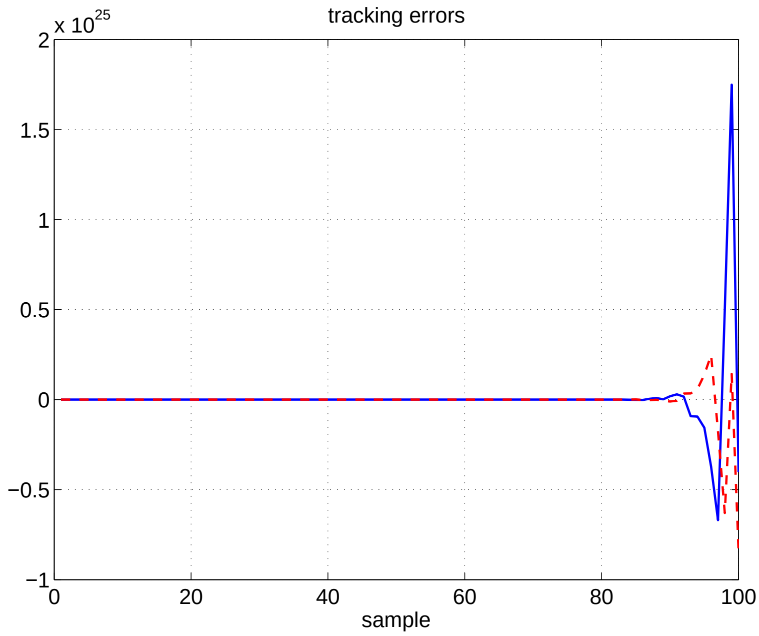

Figure 2.8: Tracking errors for discrete-time plant with $\beta < \beta_{min}$

unstable.

$$G(z) = \begin{bmatrix} \frac{4}{(z+0.4)(z-0.2)} & \frac{-2}{(z+0.4)(z-0.2)} \\ \frac{-1}{z+1.2} & \frac{1}{z+1.2} \end{bmatrix} \tag{2.49}$$

The reference model is chosen as

$$W_m(z) = \begin{bmatrix} \frac{1}{(z+0.2)^2} & 0 \\ 0 & \frac{1}{z+0.4} \end{bmatrix} \tag{2.50}$$

with reference signals $r_1 = \sin(kT)$ and $r_2 = \sin(3kT)$. The matrix gains are set to $\Gamma_1 = 2I_{8\times 8}$, $\Gamma_2 = 2I_{2\times 2}$; I/O state filters (1.29): $\Lambda(s) = \frac{1}{z+0.2}$; (observability index) $\nu = 2$; $h(s) = \frac{1}{(z+0.4)^2}$. The plant initial conditions are $x = [5\ 5\ 5]$. The controller is designed assuming that there is no prior knowledge on the plant parameters, so that the estimated parameters initial conditions are set to zero. The remaining system initial conditions are also set to zero.

Unlike the CT case, the parameter β now plays an essential role and has to be consciously chosen. We refer to (2.38) to obtain a lower bound on β:

$$\beta > 4.733$$

The result obtained using $\beta = 4$ is seen in Fig. 2.8, where it is possible to note an unstable behavior. It is interesting to note, however, that this estimate is a lower bound that guarantees stability, such that convergence is possibly achieved with values of β slightly below β_{min}.

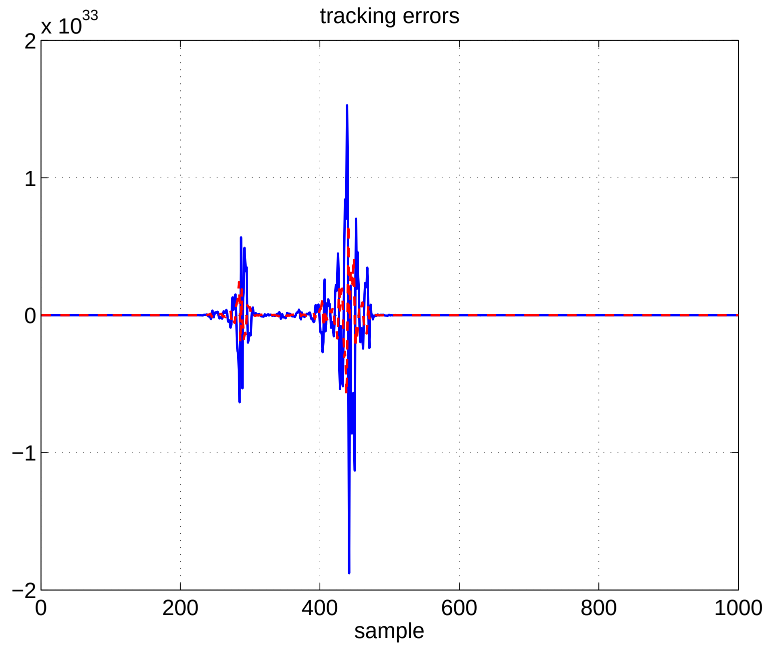

Figure 2.9: Tracking errors for discrete-time plant with $\beta = 4.75$

With $\beta = 4.75$, convergence is obtained after a large transient oscillation, as shown in Fig. 2.9, indicating that β should be tuned up the minimum value.

The transient is fairly improved with $\beta = 15$, as shown in Fig. 2.10. Tracking performance is shown in Fig. 2.11, where it is possible to note a reasonable output tracking result.

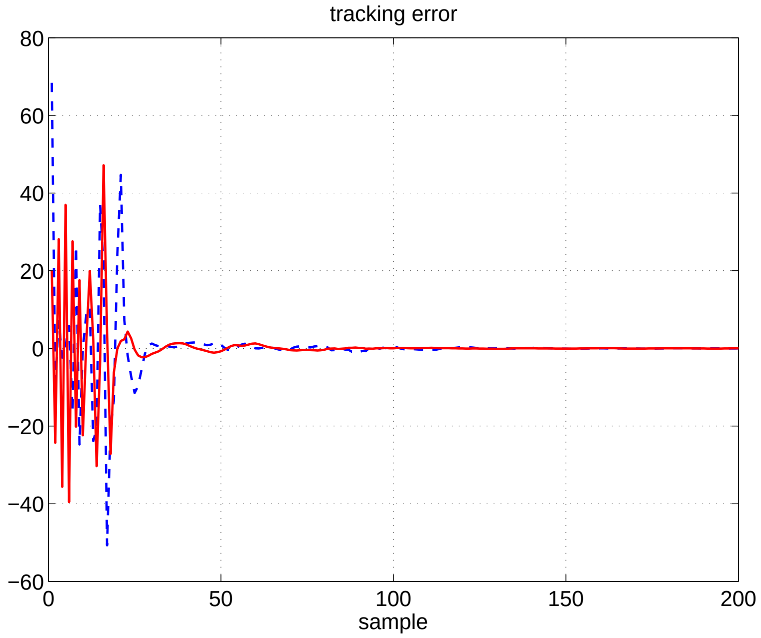

Figure 2.10: Tracking errors for discrete-time plant with $\beta = 15$

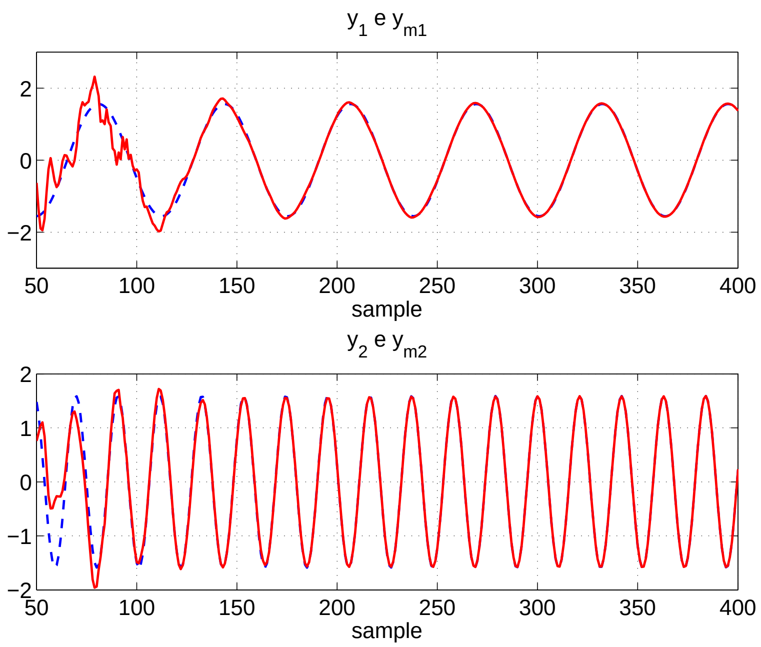

Figure 2.11: Tracking performance for discrete-time plant with $\beta = 15$

Chapter 3

Multivariable Binary-MRAC using Global Exact Differentiators (GRED-BMRAC)

3.1 Introduction

In this chapter we propose a robust controller with guaranteed transient performance for plants with arbitrary relative degree that does not require symmetry assumptions on the HFG.

This is achieved by means of a further extension to MIMO BMRAC to encompass non-uniform arbitrary relative degree plants that achieves global exact tracking by using a hybrid estimation scheme recently generalized to a multivariable framework [68]. Such estimator, named Global Robust Exact Differentiator (GRED), switches between a standard MIMO lead filter and a nonlinear one which utilizes Robust Exact Differentiators (RED) [69] based on higher order sliding modes. The use of MIMO GRED renders the error system uniformly globally exponentially practically stable with respect to a small residual set with ultimate convergence to zero.

The BMRAC (Binary Model Reference Adaptive Control) [32] is based on adaptation laws with projection. This allows the adaptive system to be tuned by increasing the adaptation gain, which makes BMRAC tend to behave as a sliding mode controller. However, such gain can be tuned up to a sufficiently large value while avoiding chattering. This allows us to obtain a better transient behavior and robustness in comparison to adaptive controllers based on gradient adaptation. This technique is also based on the PDJ condition on K_p instead of symmetry conditions. If K_p is not PDJ, we use a multiplier $\bar{L}$ such that $\bar{L}K_p$ is PDJ. Only Continuous-Time systems are considered in this case.

The formulation is the same as presented in Section 1.2 and we only deal with

CT systems. An important modification is the choice of the reference model, which is of the form

$$y_m = W_m(s)\, r\,; \qquad r, y_m \in \mathbb{R}^M \tag{3.1}$$

$$W_m(s) = diag\left\{(s+a)^{-1}, ..., (s+a)^{-1}\right\} L^{-1}(s) \tag{3.2}$$

where $a > 0$ and the poles are repeated. Also, L(s) is given by

$$L(s) = diag\left\{L_1(s), L_2(s), ..., L_M(s)\right\}, \tag{3.3}$$

where $L_i(s), i = 1, ..., M$ are Hurwitz polynomials given by

$$L_i(s) = s^{(\rho_i - 1)} + l^{[i]}_{\rho_i - 2} s^{(\rho_i - 2)} + ... + l^{[i]}_1 s + l^{[i]}_0 \tag{3.4}$$

where ρ_i is the relative degree of output i and $l_i > 0$.

The choice of the reference model follows the idea of reducing an arbitrary relative degree problem to one with uniform relative degree one, which is achieved through differentiation of output signals.

The transfer matrix $W_m(s)$ has the same vector relative degree as $G(s)$ and its HFG is the identity matrix. If K_p is PDJ, it is also possibe to conclude that the error system of Eq. (1.37) is WSPR since the model is a multiple of the identity matrix $L(s)W_m(s) = \frac{1}{s+a}I$. To consider more general reference models one could use the WASPR concept [39, 40], as in [30][1]. However, if the PDJ condition is not satisfied on K_p, it is possible to use a stabilizing multiplier $\bar{L}$ such that $\bar{L}K_p$ is PDJ.

The modified tracking error is, in this case

$$e_L(t) = \bar{L}(y(t) - y_m(t)) \tag{3.5}$$

and can be described by the following model

$$\dot{x}_e = A_c x_e + B_c K_p \left[u - u^*\right], \quad e_L = \bar{L} H_0 x_e \tag{3.6}$$

which can also be rewritten in input-output form as

$$e_L = W_m(s)\bar{L}K_p \left[u - u^*\right] \tag{3.7}$$

The BMRAC extension to MIMO systems is used as proposed in [30]. We adopt

[1]The tracking of more general reference models could be obtained by simply preshaping the reference signal r through a precompensator at the input of the above model.

the following parametrization

$$\vartheta = vec(\theta) = \begin{bmatrix} \theta^{[1]} \\ \theta^{[2]} \\ \vdots \\ \theta^{[n]} \end{bmatrix}, \quad \Omega = I_M \otimes \omega = \begin{bmatrix} \omega & & \\ & \ddots & \\ & & \omega \end{bmatrix} \tag{3.8}$$

with $\Omega \in \mathbb{R}^{NM \times M}, \vartheta \in \mathbb{R}^{NM}$, where N is the number of elements of the regressor vector ω, $\theta^{[i]}$ is the i-th column of the parameter matrix θ and $\otimes$ is the Kronecker product. The BMRAC MIMO adaptation law is given by

$$\dot{\vartheta} = -\vartheta\sigma - \gamma\Omega e_L \tag{3.9}$$

with σ given by a projection

$$\sigma = \begin{cases} 0, & \text{if} \quad \|\vartheta\| < M_\vartheta \quad \text{or} \quad \sigma_{eq} < 0 \\ \sigma_{eq}, & \text{if} \quad \|\vartheta\| \geq M_\vartheta \quad \text{and} \quad \sigma_{eq} \geq 0 \end{cases} \tag{3.10}$$

$$\sigma_{eq} = \frac{-\gamma\vartheta^T \Omega e_L}{\|\vartheta\|^2} \tag{3.11}$$

where $M_\vartheta > \|\vartheta^*\|$. The control law can be rewritten as

$$u(t) = \theta^T(t)\omega(t) = \Omega^T(t)\vartheta(t) \tag{3.12}$$

This approach is already established for plants with uniform relative degree [30]. An extension for arbitrary relative degree can be obtained using the derivatives of y such that a system with relative degree one is rendered.

To provide some insight in this idea, we consider the following plant

$$y = \begin{bmatrix} \frac{1}{(s+1)(s+2)} & 0 \\ 0 & \frac{1}{s+3} \end{bmatrix} u$$

which has a nonuniform relative degree ($\rho_1 = 2, \rho_2 = 1$). The underlying idea of the GRED-BMRAC is to generate a system with uniform relative degree one through output derivatives, such that a following modified output is obtained. In this illustrative example, we would have

$$\xi_y = \begin{bmatrix} s+1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{(s+1)(s+2)} & 0 \\ 0 & \frac{1}{s+3} \end{bmatrix} u \tag{3.13}$$

Which is equivalent to

$$\xi_y = L(s)y = \dot{y} + y$$

Instead of using the tracking error $e = y - y_m$, we use a modified error of relative degree one

$$\bar{e} = \xi_y - \xi_m \tag{3.14}$$

Thus, to overcome the relative degree restriction, we employ the operator defined in Eq. (3.4) such that $L(s)\bar{G}(s)$ and $L(s)W_m(s)$ have uniform vector relative degree one. Note that since $W_m(s)$ is chosen by design, ξ_m is easily generated without the need of calculating derivatives.

However, we still have to address the problem of obtaining ξ_y since it is not directly available as the operator of Eq. (3.13) is not implementable. To solve this problem, we use a global robust exact differentiator (GRED) that switches between a lead filter and a nonlinear estimator. The GRED provides exact derivatives, such that a modified system of relative degree one is then rendered and BMRAC can be applied.

3.2 BMRAC using a MIMO lead filter

The BMRAC proposed in [30] achieves global exact tracking if the considered plant has uniform relative degree one.

To overcome the relative degree restriction, we employ the operator defined in Eq. (3.4) such that $L(s)G(s)$ and $L(s)W_m(s)$ have uniform vector relative degree one. To this end we define the following modified output

$$\xi_y = L(s)y = \begin{bmatrix} y_1^{(\rho_1-1)} + \cdots + l_1^{[1]}\dot{y}_1 + l_0^{[1]}\; y_1 \\ \vdots \\ y_M^{(\rho_M-1)} + \cdots + l_1^{[M]}\dot{y}_M + l_0^{[M]} y_M \end{bmatrix} = \begin{bmatrix} \sum_{j=0}^{\rho_1-1} l_j^{[1]} h_1^T A_c^{(j)} X \\ \vdots \\ \sum_{j=0}^{\rho_M-1} l_j^{[M]} h_M^T A_c^{(j)} X \end{bmatrix} = \bar{H}X\,, \tag{3.15}$$

where $h_i^T \in \mathbb{R}^{n+2M(\nu-1)}$ is the i-th ' of matrix H_o and the second equality is derived from Assumption (A4) and (3.6). We create a modified WSPR error of relative degree one:

$$\bar{e}_L = \bar{L}(\xi_y - \xi_m); \quad \xi_m = L(s)y_m; \tag{3.16}$$

$$\dot{x}_e = A_c x_e + B_c K_p\,[u - u^*]\,, \quad \bar{e}_L = \bar{L}H_0 x_e \tag{3.17}$$

which can also be rewritten in input-output form as follows, since $W_m(s)$ commutes with $\bar{L}$

$$\bar{e}_L = W_m(s)\bar{L}K_p\,[u - u^*] \tag{3.18}$$

Note that ξ_m is directly available for implementation while the signal ξ_y needed to overcome the relative degree obstacle is not A possible way to solve this problem is to estimate ξ_y by means of a lead filter.

$$\hat{\xi}_l = L_a(s)\xi_y,\ L_a(s) = L(s)F^{-1}(\tau s) \tag{3.19}$$

where $F(\tau s)=\text{diag}\{(\tau s+1)^{\rho_1-1},\ldots,(\tau s+1)^{\rho_m-1}\}$. One can note that as $\tau>0$ tends to zero, $\hat{\xi}_l$ approximates ξ_y. The error signal that will drive adaptation is an estimate of $\bar{e}_L(t)$:

$$\hat{\bar{e}}_L = \bar{L}(\hat{\xi}_l - \xi_m) \tag{3.20}$$

Defining the lead filter estimation error as the difference between the estimate of ξ_y obtained by the lead filter and its actual value

$$\varepsilon_l = \hat{\xi}_l - \xi_y \tag{3.21}$$

its dynamics can be described by:

$$\dot{x}_\varepsilon = \frac{1}{\tau}A_\varepsilon x_\varepsilon + B_\varepsilon\dot{\xi}_y, \quad \varepsilon_l = H_\varepsilon x_\varepsilon\,, \tag{3.22}$$

where $\dot{\xi}_y = \bar{H}A_cX + \bar{H}B_cK_p\tilde{\vartheta}^T\Omega + \bar{H}B_cr$ (see (1.34) and (3.15)), $A_\varepsilon = \text{block diag}\{A_\varepsilon^{[1]},\ldots,A_\varepsilon^{[M]}\}$, $B_\varepsilon = \text{block diag}\{B_\varepsilon^{[1]},\ldots,B_\varepsilon^{[M]}\}$, $H_\varepsilon = \text{block diag}\{H_\varepsilon^{[1]},\ldots,H_\varepsilon^{[M]}\}$, with$A_\varepsilon^{[i]} \in \mathbb{R}^{\rho_i-1\times\rho_i-1}$, $B_\varepsilon^{[i]} \in \mathbb{R}^{\rho_i-1\times 1}$, $H_\varepsilon^{[i]} \in \mathbb{R}^{1\times\rho_i-1}$,

$$\begin{aligned}
A_\varepsilon^{[i]} &= \begin{bmatrix} -a_{\rho_i-2}^{[i]} & 1 & 0 & \ldots & 0 \\ -a_{\rho_i-3}^{[i]} & 0 & 1 & \ldots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -a_1^{[i]} & 0 & 0 & 0 & 1 \\ -a_0^{[i]} & 0 & 0 & 0 & 0 \end{bmatrix}, B_\varepsilon^{[i]} = \begin{bmatrix} -b_{\rho_i-2}^{[i]} \\ -b_{\rho_i-3}^{[i]} \\ \vdots \\ -b_1^{[i]} \\ -b_0^{[i]} \end{bmatrix}, \\
H_\varepsilon^{[i]} &= \begin{bmatrix} 1 & 0 & 0 & \ldots & 0 \end{bmatrix}, \\
a_j^{[i]} &= \mathcal{C}_{\rho_i-1-j}^{\rho_i-1}, \quad b_j^{[i]} = \mathcal{C}_{j+1}^{\rho_i-1}\ \ \mathcal{C}_l^n = n!/(k!(n-k)!)
\end{aligned}$$

In the stability analysis of the closed-loop error system, with state $z^T = \begin{bmatrix} x_e^T\ x_\varepsilon^T \end{bmatrix}$, we will consider the presence of a uniformly bounded output disturbance $\beta_\alpha(t)$ of order τ. Taking into account the presence of $\beta_\alpha(t)$, the lead estimation error can be represented as the :

$$\varepsilon_l = \hat{\xi}_l - \xi_y + \beta_\alpha(t) \tag{3.23}$$

where by design $\beta_\alpha(t) \le \varepsilon_M$ with $\varepsilon_M = \tau K_R$

Using the MIMO lead filter the adaptation law is given by

$$\dot{\vartheta} = -\vartheta\sigma - \gamma\Omega(\hat{\bar{e}}_L + \beta_\alpha) \tag{3.24}$$

with σ given by a projection

$$\sigma = \begin{cases} 0, & \text{if} \quad \|\vartheta\| < M_\vartheta \quad \text{or} \quad \sigma_{eq} < 0 \\ \sigma_{eq}, & \text{if} \quad \|\vartheta\| \geq M_\vartheta \quad \text{and} \quad \sigma_{eq} \geq 0 \end{cases} \tag{3.25}$$

$$\sigma_{eq} = \frac{-\gamma\vartheta^T\Omega(\hat{\bar{e}}_L + \beta_\alpha)}{\|\vartheta\|^2} \tag{3.26}$$

where $M_\vartheta > \|\vartheta^*\|$. At this point, the following Theorem can be stated.

Theorem 3 *Consider the plant (1.21) and the reference model (3.1)–(3.3) with control signal (3.12) and adaptation law (3.24)–(3.26). Suppose that assumptions (A1) to (A5) hold. If the disturbance $\beta_\alpha(t)$ is uniformly bounded by $\|\beta_\alpha(t)\| \leq \tau K_R$, where $K_R > 0$ is a constant, then for sufficiently small $\tau > 0$ and sufficiently large $\gamma > 0$, the closed-loop error system (3.6), (3.12), (3.15), (3.22), (3.24)–(3.26) with state $z^T = \begin{bmatrix} x_e^T \; x_\varepsilon^T \end{bmatrix}$, is uniformly globally exponentially practically stable (GEpS) with respect to a residual set, i.e., there exist constants $c_z, a > 0$ such that $\|z(t)\| \leq c_z e^{-a(t-t_0)}\|z(t_0)\| + \mathcal{O}(\sqrt{\tau}) + \mathcal{O}(\sqrt{1/\gamma})$ holds $\forall z(t_0)$, $\forall t \geq t_0 > 0$.*

(Proof: see Appendix B.1)

Corollary 1 *For all $R > 0$, there exists $\tau > 0$ sufficiently small and γ sufficiently large such that for some finite time T, the error state $z(t)$ is steered to an invariant compact set $D_R := \{z : \|z\| \leq R\}$.*

Corollary 2 *Signals $y_j^{(i)}(t), i = 0,\ldots,\rho_j,\; j = 1,\ldots,M$ are uniformly bounded, i.e., $\exists K_i^{[j]} > 0$ such that $|y_j^{(i)}(t)| \leq K_i^{[j]}, \forall\, t \geq t_0 \geq 0,\; i=0,\ldots,\rho_j,\; j=1,\ldots,M$. Moreover, if $\|x_e(t)\| \leq R, \forall t > T$, then, $\exists C_{\rho_j}^{[j]} > 0$ such that $\left\| y_{j_{[T,t]}}^{(\rho_j)} \right\|_\infty \leq C_{\rho_j}^{[j]}, j = 1, \ldots, M$. (Proof: see Appendix B.2)*

3.3 MIMO Robust Exact Differentiators

A possible extension of the RED to MIMO systems could be obtained using an RED of appropriate order in each output. The order of differentiators in each output is chosen so as to to obtain outputs or relative degree one.

The idea is to use a RED of order $p_j = \rho_j - 1$ for each output $y_j \in \mathbb{R},\; j = 1, \ldots, M$.

$$
\begin{cases}
\dot{\zeta}_0^{[j]} = v_0^{[j]}, \; v_0^{[j]} = -\lambda_0^{[j]} \left| \zeta_0^{[j]} - y_j(t) \right|^{\frac{p_j}{p_j+1}} \operatorname{sgn}(\zeta_0^j - y_j(t)) + \zeta_1^{[j]} \\
\dot{\zeta}_1^{[j]} = v_1^{[j]}, \\
v_1^{[j]} = -\lambda_1 \left| \zeta_1^{[j]} - v_0^{[j]} \right|^{(n-1)/n} sign(\zeta_1^{[j]} - v_0^{[j]}) + \zeta_2^{[j]} \\
\quad \vdots \\
\dot{\zeta}_i^{[j]} = v_i^{[j]}, \\
v_i^{[j]} = -\lambda_i^{[j]} C_{\rho_j}^{[j]\,\frac{1}{p_j-i+1}} \left| \zeta_i^{[j]} - v_{i-1}^{[j]} \right|^{\frac{p_j-i}{p_j-i+1}} = \operatorname{sgn}(\zeta_i^{[j]} - v_{i-1}^{[j]}) + \zeta_{i+1}^{[j]}, \\
\quad \vdots \\
\dot{\zeta}_{p_j}^{[j]} = -\lambda_{p_j}^{[j]} C_{\rho_j}^{[j]} \operatorname{sgn}(\zeta_{p_j}^{[j]} - v_{p_j}^{[j]}),
\end{cases}
\tag{3.27}
$$

where $i = 0, \ldots, p_j - 1$, $v_{-1}^{[j]} = y_j(t)$, $C_{\rho_j}^{[j]}$ is a known constant such that $|y_j^{(\rho_j)}(t)| \leq C_{\rho_j}^{[j]}$, $\forall t$. If the parameters $\lambda_i^{[j]}$ are properly recursively chosen then the equalities

$$
\zeta_0^{[j]} = y_j(t); \; \zeta_i^{[j]} = y_j^{(i)}(t), \quad j = 1, \ldots, M; \quad i = 1, \ldots, p_j
$$

are established in finite time [69].

Under the foregoing conditions, the above differentiator can provide the exact $y_j(t)$ derivatives. According to [69], the RED's performance is asymptotically optimal in the presence of small Lebesgue-measurable input noise. Moreover, it is important to stress that the variables of each individual RED (3.27) cannot escape in finite time. This result is formalized in the following Lemma.

Lemma 10 *Consider system (3.27), with state* $\zeta^{[j]} = [\zeta_0^{[j]} \; \ldots \; \zeta_{p_j}^{[j]}]^T$. *If* $\left| y_j^{(i)}(t) \right| \leq K_i^{[j]}$, $i = 0, \ldots, \rho_j$, $\forall t$ *(finite), for some positive constants* $K_i^{[j]}, i = 0, \ldots, \rho_j$, *then* $\zeta^{[j]}(t)$ *cannot diverge in finite time.*

(Proof: see [68]) ■

Thus, using a MIMO RED, composed by M REDs of order $\rho_j - 1$ for each output y_j, the following estimate for ξ_y can be obtained:

$$
\hat{\xi}_r = \begin{bmatrix} \zeta_{\rho_1-1}^{[1]} + \cdots + l_1^{[1]} \zeta_1^{[1]} + l_0^{[1]} \zeta_0^{[1]} \\ \vdots \\ \zeta_{\rho_M-1}^{[M]} + \cdots + l_1^{[M]} \zeta_1^{[M]} + l_0^{[M]} \zeta_0^{[M]} \end{bmatrix}. \tag{3.28}
$$

Then, the derivatives of y could be used as in $\xi_y = L(s)y$. However, only local convergence of the error state to zero could be guaranteed, since the signals $y_j^{(\rho_j)}(t)$, $j = 1, \ldots, M$ should be uniformly bounded.

Using the MIMO RED the adaptation law is given by

$$\dot{\vartheta} = -\vartheta\sigma - \gamma\Omega\bar{L}(\hat{\xi}_r - \xi_m) \tag{3.29}$$

with σ given by a projection

$$\sigma = \begin{cases} 0, & \text{if} \quad \|\vartheta\| < M_\vartheta \quad \text{or} \quad \sigma_{eq} < 0 \\ \sigma_{eq}, & \text{if} \quad \|\vartheta\| \geq M_\vartheta \quad \text{and} \quad \sigma_{eq} \geq 0 \end{cases} \tag{3.30}$$

$$\sigma_{eq} = \frac{-\gamma\vartheta^T\Omega\bar{L}(\hat{\xi}_r - \xi_m)}{\|\vartheta\|^2} \tag{3.31}$$

where $M_\vartheta > \|\vartheta^*\|$.

It is important to note that the control input (3.12) with adaptation law (3.29)–(3.31) guarantees only local/semi-global stability properties, since the RED requires that $y^{(\rho_i)}$ should be uniformly bounded to provide exact derivatives.

3.4 Global RED based BMRAC

The Global RED is a hybrid compensator which consists of a (time-varying) convex combination of a lead filter estimate (3.19) and a RED estimate (3.28) according to:

$$\hat{\xi}_g = \alpha(\tilde{\nu}_{rl})\ \hat{\xi}_l + [1 - \alpha(\tilde{\nu}_{rl})]\ \hat{\xi}_r\,, \tag{3.32}$$

where $\tilde{\nu}_{rl} = \hat{\xi}_r - \hat{\xi}_l$ is the difference between both estimates. The switching function $\alpha(\tilde{\nu}_{rl})$ is a continuous, state dependent modulation which assumes values in the interval $[0, 1]$ and allows the controller to smoothly change from one estimator to the other.

Specifically, $\alpha(\cdot)$ is designed such that $\left\|\hat{\xi}_g - \hat{\xi}_l\right\| \leq \tau K_R$:

$$\alpha(\tilde{\nu}_{rl}) \begin{cases} 0, & \|\tilde{\nu}_{rl}\| < \varepsilon_M - \Delta \\ (\|\tilde{\nu}_{rl}\| - \varepsilon_M + \Delta)/\Delta, & \varepsilon_M - \Delta \leq \|\tilde{\nu}_{rl}\| < \varepsilon_M \\ 1, & \|\tilde{\nu}_{rl}\| \geq \varepsilon_M \end{cases} \tag{3.33}$$

where $0 < \Delta < \varepsilon_M$ is a boundary layer used to smoothen the switching function, and $\varepsilon_M := \tau K_R$ with K_R being an appropriate positive design parameter, that is selected such that $\varepsilon_M - \Delta > \bar{\varepsilon}_l$. This implies that after some finite time only the MIMO RED is active ($\alpha = 0$), providing exact estimation of the output derivatives ξ_y, as desired. Some insight on how to tune MIMO GRED parameters is given below (for further refereence, see [68]).

Remark 3 *The parameter ε_M should be small enough in order to guarantee that* *the MIMO RED is used only when it provides a satisfactory derivative estimate, otherwise a poor transient tracking performance could result. On the other hand if ε_M is not large enough, then the switching scheme could not ultimately select the MIMO RED, thus degrading the steady state tracking performance. The parameter Δ is only used to smoothly switch between the MIMO RED and the MIMO lead filter. As well as the boundary layer method, it may be advantageous in practice in order to reduce noise sensitivity.*

In order to guarantee global exponential stability with respect to a small residual set and to achieve global convergence of the error state to zero, we show that the BMRAC using a MIMO lead filter presented in Section 3.2 can be combined with the MIMO RED (Section 3.3).

It should be noted that global stability to an invariant compact set D_R is guaranteed independently of switching between both estimators since it is possible to show that the resulting system is equivalent to a BMRAC using a MIMO lead filter with a uniformly bounded output disturbance of order τ. Thus, global practical stability and convergence to the compact set D_R are guaranteed, according to Theorem 3. The switching function is properly chosen to ensure that after some finite time only the estimate provided by the MIMO RED is used.

Using the GRED to estimate ξ_y the adaptive law is

$$\dot{\vartheta} = -\vartheta\sigma - \gamma\Omega\bar{L}(\hat{\xi}_g - \xi_m) \tag{3.34}$$

with σ given by a projection

$$\sigma = \begin{cases} 0, & \text{if} \quad \|\vartheta\| < M_\vartheta \quad \text{or} \quad \sigma_{eq} < 0 \\ \sigma_{eq}, & \text{if} \quad \|\vartheta\| \geq M_\vartheta \quad \text{and} \quad \sigma_{eq} \geq 0 \end{cases} \tag{3.35}$$

$$\sigma_{eq} = \frac{-\gamma\vartheta^T\Omega\bar{L}(\hat{\xi}_g - \xi_m)}{\|\vartheta\|^2} \tag{3.36}$$

where $M_\vartheta > \|\vartheta^*\|$. The stability and convergence results of the proposed control scheme are stated in the following theorem. A block diagram of such scheme is shown in Fig. 3.1.

Theorem 4 *Consider the plant (1.21) and the reference model (3.1)–(3.3) with control law given by (3.12) and adaptation law (3.34)–(3.36). The switching function $\alpha(\cdot)$ is defined in (3.33). Suppose that assumptions (A1) to (A5) hold. with $\vartheta(0) <= M_\vartheta$ and for sufficiently small $\tau > 0$ and sufficiently large $\gamma > 0$, the closed-loop error system described by (3.12), (3.6), (3.15), (3.22), (3.34)–(3.36) is uniformly globally exponentially practically stable (GEpS) with respect to a residual set and the MIMO*

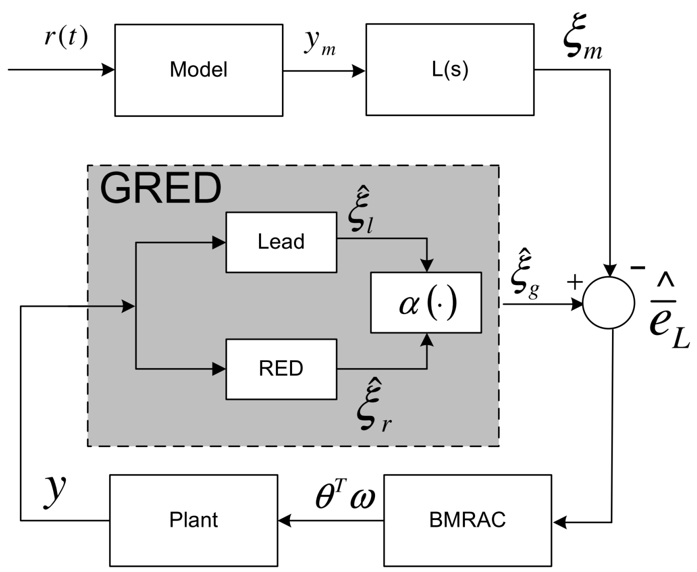

Figure 3.1: Block diagram of GRED-BMRAC

RED estimation and all closed-loop signals are uniformly bounded. Moreover, for $\lambda_i^{[j]}$*,* $j = 1, \ldots, M$*,* $i = 0, \ldots, \rho_j - 1$*, and* K_R *properly chosen, the estimation of the output derivatives* ξ_y *becomes exact, being made exclusively by the RED (*$\alpha(\cdot) = 0$*) after some finite time. Then, the closed-loop error state* $z^T = \left[x_e^T \; x_\varepsilon^T\right]$*, and hence the output tracking error* e*, converge exponentially to zero. (Proof: see Appendix B.3)*

3.5 Simulation Results

Consider a MIMO LTI actuator/process similar to the example used in [51] described by

$$A = \begin{bmatrix} 1 & 2 & 3 & 1 \\ 0 & 2 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}; \; B = \begin{bmatrix} 10 & 20 & 0 & 0 \\ -1 & -0.5 & 0 & -0.5 \\ 0 & 0 & 16 & 80 \\ 0 & 0 & 0 & 4 \end{bmatrix},$$

$$H = I$$

which has transfer matrix $G_0(s)$. The plant is composed by this actuator/process and a sensor with transfer matrix given by $G_s(s) = diag\{1/(s+1), 1/(s+1), 1, 1\}$. The resulting transfer matrix $G(s) = G_s(s)G_0(s)$ from u to y has the following vector relative degree ($\rho_1 = 2, \rho_2 = 2, \rho_3 = 1, \rho_4 = 1$). The model is chosen as $W_m(s) = diag\{1/(s+1), 1/(s+1), 1/(s+1), 1/(s+1)\}$. For simplicity, we have chosen $K_p = B$ to be PDJ. Then, with $L(s) = \text{diag}\,\{(s+1), (s+1), 1, 1\}$, the corresponding transfer matrix $L(s)W_m(s)K_P$ *can be shown to be WSPR*. When this is not the case, a passivating multiplier can be used based on some nominal K_p [40]. According

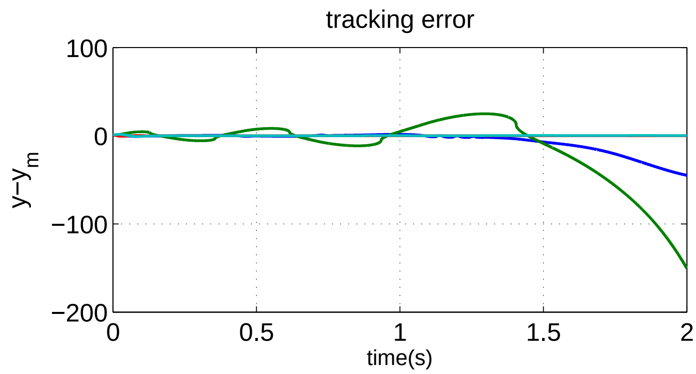

Figure 3.2: Tracking errors obtained with BMRAC using only the RED to compensate relative degree

to the proof of the WASPR Theorem of [39], it is easy to conclude that a WSPR system will remain WSPR with any output feedback with static gain $-k$, where k is a positive scalar. It was verified that one could improve the convergence speed of the tracking error by tuning up the scalar k.

The reference input signal $r(t) \in \mathbb{R}^4$ was chosen as biased square waves $f(t/T) = sqw(2t/T)+1$ with different periods T, $r^T = 0.5[f(t/6); 2f(t/2); 3f(t/4); 3f(t/0.3)]$. The only prior knowledge needed for the control design is $M_\vartheta = 15$ (see (3.36)) and observability index $\nu = 2$. The adaptation gain is $\gamma = 10$. Other design parameters are: Lead filter (3.19): $\tau = 0.01s$; MIMO RED (3.27)-(3.28): $\lambda_0^{[1]} = 1.5{C_2^{[1]}}^{1/2}$, $\lambda_1^{[1]} = 1.1C_2^{[1]}$ and $C_2^{[1]} = 10$; switching function (3.33): $\varepsilon_M = 0.5$ and $\Delta = 0.2$. The plant initial conditions are $y(0) = [1\ 1\ 1\ 1]$. The remaining system initial conditions are set to zero.

Using only the RED, the system shows unstable behavior, as seen in Fig. 3.2. When only the lead filter is used, large error persists as shown in Fig. 3.3. When the hybrid differentiator is employed, zero tracking errors are achieved within numerical integration errors, as seen in Fig. 3.4. Tracking performance is shown in Fig. 3.5. Control chattering was also avoided as shown in Fig. 3.6. Finally, in Fig. 3.7 one can note that the differentiation is firstly done by the lead filters and it switches permanently to the MIMO RED in finite time. It should be noted that under the same conditions, the system is unstable if only the MIMO RED is used.

3.5.1 Comparison with Bilinear MRAC

Revisiting the example of the process control plant in Subsection 2.3.1, it is possible to illustrate the advantages of the Binary-MRAC.

In this example, the references are square waves $r(t) = [5sqw(t/3) \quad 5sqw(t)]$. The performance obtained with the Blinear MRAC in Chapter 2 is shown on Fig. 3.8–3.9 where it is possible to note a slow convergence of the tracking error.

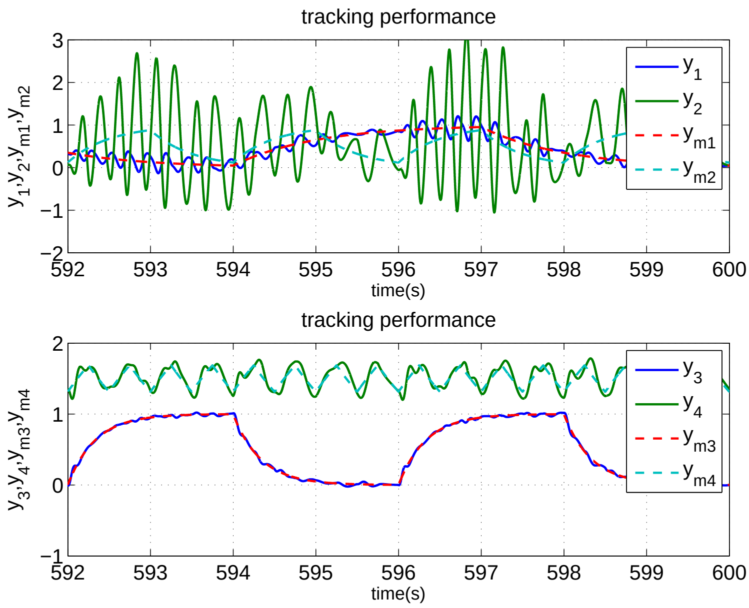

Figure 3.3: Tracking errors obtained with BMRAC using only the lead filter to compensate relative degree

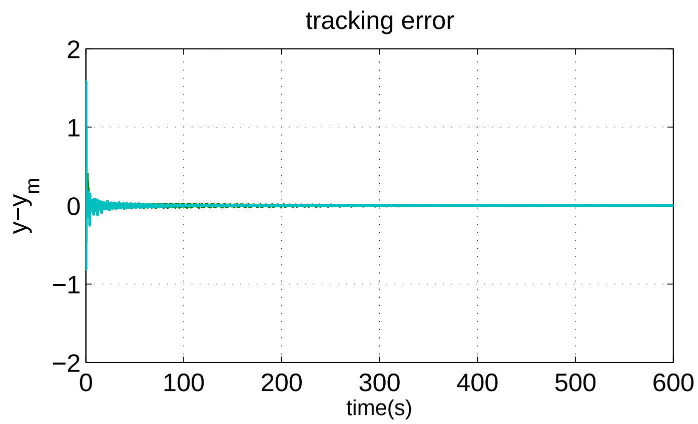

Figure 3.4: Tracking errors obtained with the hybrid scheme (GRED-BMRAC)

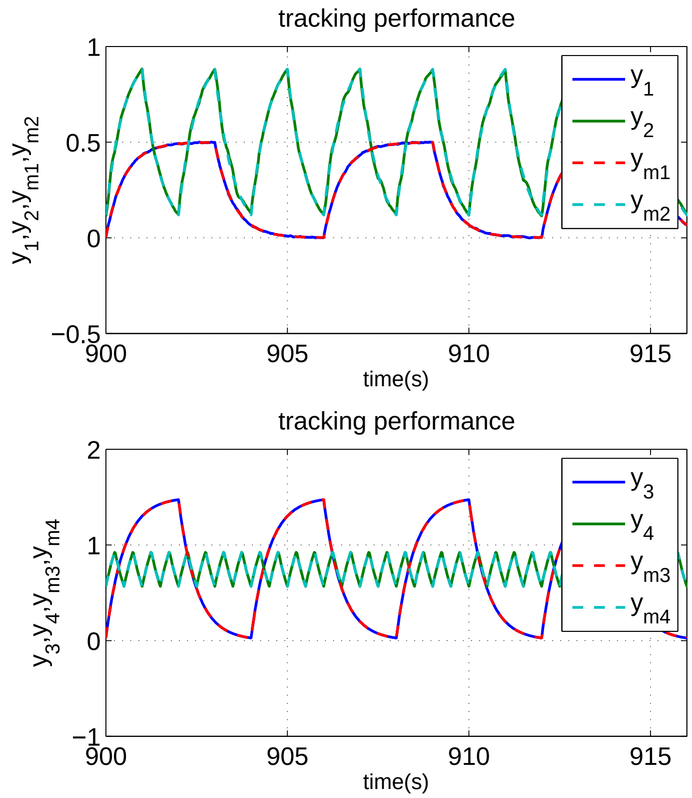

Figure 3.5: Tracking performance obtained with the hybrid scheme (GRED-BMRAC)

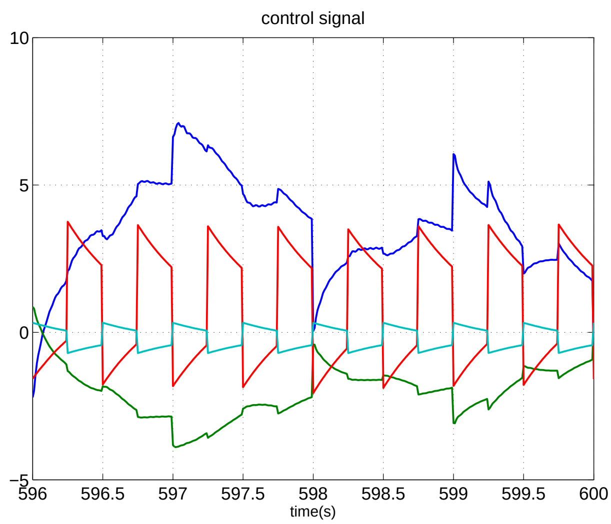

Figure 3.6: Control signal of the hybrid scheme (GRED-BMRAC)

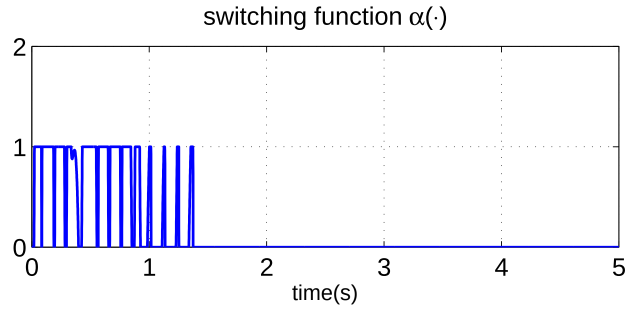

Figure 3.7: Switching function of the hybrid scheme (GRED-BMRAC)

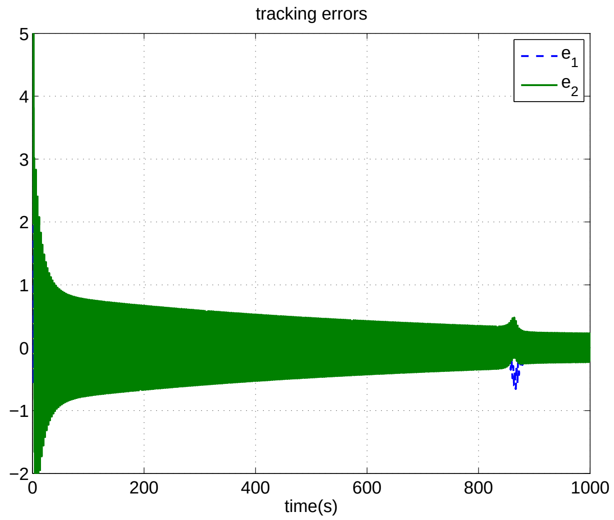

Figure 3.8: Bilinear MRAC tracking performance: plant output y (—); and model output y_m (- -)

Due to the guaranteed transient property of BMRAC, this performance is fairly improved when using GRED-BMRAC. The result is seen in Fig. 3.10–3.11.

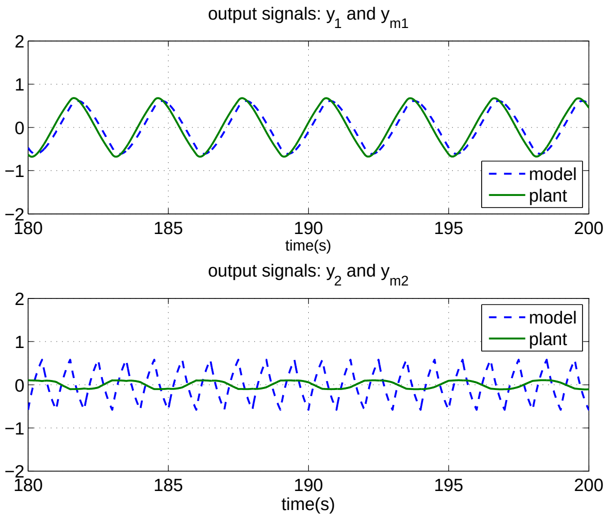

Figure 3.9: Window of Fig. 3.8. Bilinear MRAC tracking performance: plant output y (—); and model output y_m (- -)

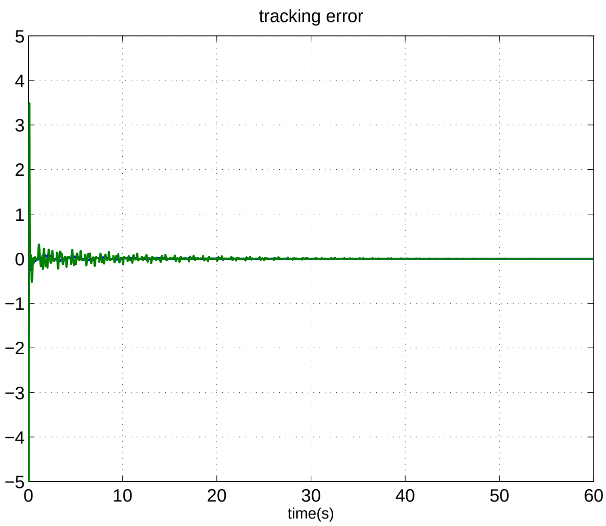

Figure 3.10: GRED-BMRAC tracking performance: plant output y (—); and model output y_m (- -)

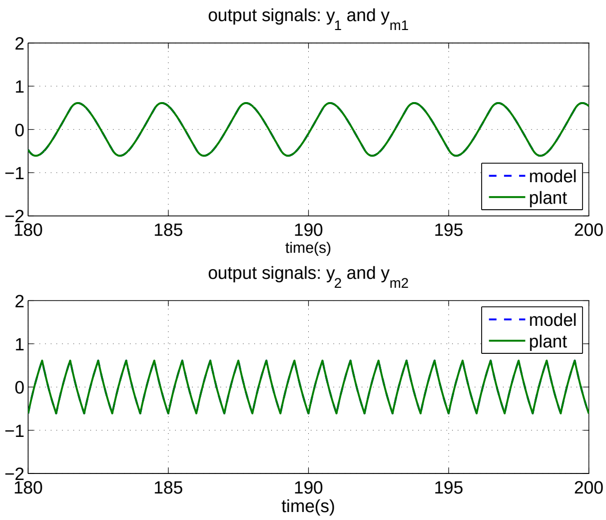

Figure 3.11: GRED-BMRAC tracking performance: plant output y (—); and model output y_m (- -)

Chapter 4

Extended Binary-MRAC

4.1 Introduction

The use of high gain adaptation laws with projection, as discussed in the previous chapter, improves robustness of adaptive controllers by ensuring parameter boundedness. We showed that the Binary-MRAC also has the desirable property of guaranteed transient, that is, tracking errors converge exponentially to a residual set with ultimate convergence to zero. The BMRAC tends to a variable structure controller as the gain tends to infinity, but the control signal is still smooth and free of chattering in the absence of unmodeled dynamics that would destroy the relaive degree one assumption.

Since we are interested in robust controllers, a technique worth of attention is the Smooth Sliding Control, proposed in [71]. It consists of a Variable Structure control with MRAC parametrization with input filtered control and the use of a prediction loop. The result is a controller that is robust to unmodeled dynamics and time-delays and shows good transient performance. Also, due to the filtering and prediction loop features, it generates a control signal that is also free of chattering.

The BMRAC and the SSC present interesting characteristics, which can be combined to generate a new controller. In this Chapter we combine the use of high gain adaptation with projection with input filtering and prediction loop. This leads to a new controller that has interesting robustness and transient properties. We call this new architecture the Extended BMRAC (eBMRAC).

The main features of the resulting combination can be also found in the recently proposed L1 Adaptive Control (L1-AC) [58]. Since this controller was widely discussed in the past few years, we dedicate a section to compare both techniques. The L1-AC is known to present some limitations in tracking time varying references, despite its robustness and performance for stabilization purposes.

We discuss this new technique mainly in the SISO framework. Preliminary results to a Multivariable extension are discussed at the end of the Chapter. We deal only with CT systems.

The control objective is to find a feedback control signal $u(t)$ for the plant (1.21) with unknown $G(s)$ and $M = 1$ such that $y(t)$ tracks $y_m(t)$ as close as possible and the closed-loop system is globally stable in the sense that all signals in the system are bounded for any bounded initial conditions and input signals.

The control input that achieves matching between model and plant is given by (1.26) with parameters defined in Eq. (1.27) and the regressor of Eq. (1.28). Tracking error is given by

$$e = y - y_m$$

whose states are

$$\dot{x}_e = A_c x_e + k^* b_c [u - u^*]; \quad e = h_c^T x_e \tag{4.1}$$

or in input/output form as

$$e = k^* W_m(s)[u - u^*]; \quad k^* = k_p / k_m \tag{4.2}$$

Since the parameter vector is not known, the control input is designed using an estimate θ of the ideal parameter θ^*. The implementable control law is given by:

$$u(t) = \theta^T(t)\omega(t) \tag{4.3}$$

4.2 Binary Model Reference Adaptive Control (BMRAC)

The Binary-MRAC for arbitrary relative degree plants was proposed in [70] and consists of a high gain projection adaptation based MRAC. The resulting system presents better transient performance and robustness to unmodeled dynamics than conventional adaptive controllers.

For the case $\rho = 1$, θ is obtained by a projection-type adaptation law

$$\dot{\theta}(t) = -\sigma(t)\theta(t) - \gamma e(t)\omega(t) \tag{4.4}$$

with high gain γ and the projection factor defined as

$$\sigma = \begin{cases} 0\,, & \text{if } \|\theta\| < M_\theta \text{ or } \sigma_{eq} < 0 \\ \sigma_{eq}\,, & \text{if } \|\theta\| \geq M_\theta \text{ and } \sigma_{eq} \geq 0 \end{cases} \tag{4.5}$$

with a constant $M_\theta \geq \|\theta^*\|$ and

$$\sigma_{eq} = \frac{-\gamma e \theta^T \omega}{\|\theta\|^2} \tag{4.6}$$

The BMRAC scheme is depicted in Fig. 4.1.

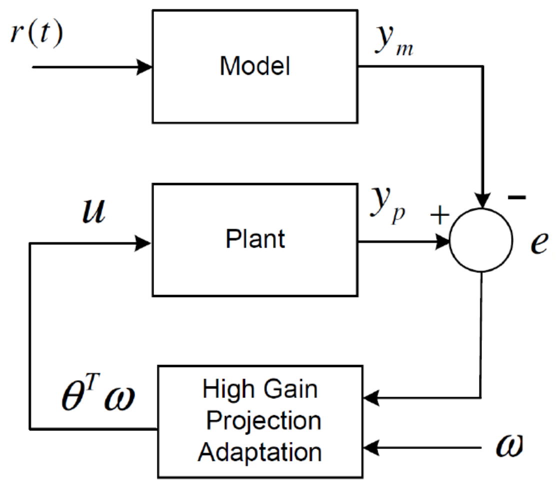

Figure 4.1: BMRAC block diagram

4.3 Smooth Sliding Control (SSC)

The Smooth-Sliding Control technique was proposed in [71] as a solution to avoid chattering in variable structure model reference control systems. The architecture of L1-AC resembles the SSC closely, but with a few key differences: i) the absence of an explicit reference model; ii) the use of projection based adaptation instead of relay switched control; iii) the need to structurally redesign the algorithm to deal with output feedback and unknown input gain; and iv) the inability to track a time-varying reference with acceptable error as will be seen in Section. 4.4. Here, we focus on the case $\rho = 1$ for the sake of simplicity, since only straightforward modifications are needed to deal with arbitrary relative degree [71].

4.3.1 Plants with relative degree one

The SSC is also based on the MRAC framework where the error equations are given by (4.1)-(4.2). The SSC employs a filtered input and a prediction loop, which is also used in the L1-AC. The smooth control law is obtained using an averaging filter with sufficiently small time constant τ such that the control u is replaced by u_0^{av}, an approximation of the equivalent control $(u_0)_{eq}$. The control law is

$$u = u^{nom} - u_0^{av}; \quad u_0^{av} = (1/F_{av}(\tau s))u_0 \tag{4.7}$$

$$u_0 = f(t)sign(\varepsilon_0) \tag{4.8}$$

where ε_0 is an output prediction error associated with the prediction loop

$$\varepsilon = e - \hat{e}; \quad \hat{e} = k^{nom}W_m(s)[u_0 - u_0^{av}]; \tag{4.9}$$

And the usual output feedback error is used as follows.

$$e = k^*W_m(s)[u - u^*]; \quad k^* = k_p/k_m \tag{4.10}$$

where $W_m(s)$ is a reference model chosen SPR. since $\hat{e}$ can be interpreted as a predicted output error by considering k^{nom} and u_0 as estimates of k^* and $u^{nom} - u^*$, respectively. With correct estimates the prediction would be exact, as seen in eq. (4.8), (4.9) and (4.2). The modulation function $f(t)$ is chosen such that $f(t) \geq |u^*(t) - u^{nom}(t)|\,; \forall t$. The SSC scheme is presented in Fig 4.2.

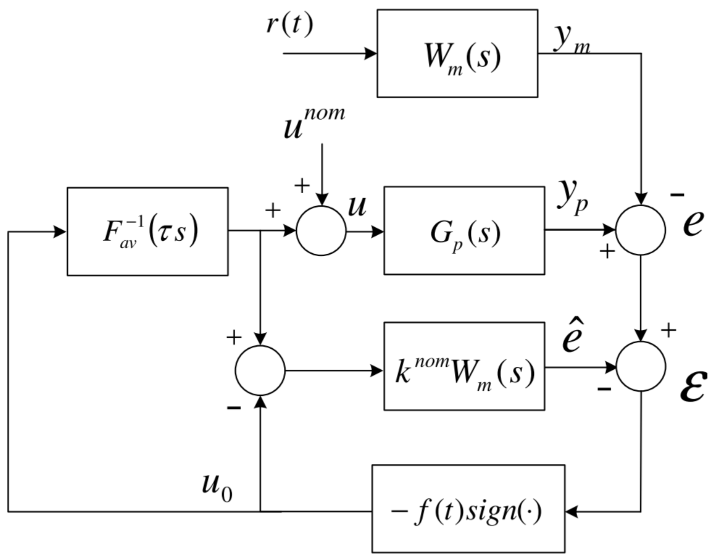

Figure 4.2: Smooth sliding control block diagram

4.4 Combining SSC and B-MRAC

The core idea of this chapter is to propose a controller that combines the SSC and BMRAC by using the SSC architecture with the BMRAC adaptation. The resulting scheme is named Extendend BMRAC (eBMRAC) To that end, the relay of the SSC is replaced by an output feedback projection adaptation law with standard MRAC parametrization. The scheme is seen in Fig. 4.3.

Consider the MRAC error equations (4.1)-(4.2). As well as the SSC, the eBMRAC uses an input filtered control signal

$$u(t) = C(s)[u_0(t)]; \quad u_0(t) = \theta(t)^T \omega(t) \tag{4.11}$$

where θ is an adaptive parameter. For the sake of simplicity, the plant high frequency gain k_p is assumed known. Nevertheless, the case where only $sign(k_p)$ is known can be addressed using a similar development as in [71].

Consider the auxiliary errors

$$\hat{e} = k^* W_m(s)[u - u_0], \quad \varepsilon = e - \hat{e} \tag{4.12}$$

The adaptation law with parameter projection is

$$\dot{\theta}(t) = -\sigma\theta - \gamma\varepsilon\omega \tag{4.13}$$

$$\sigma = \begin{cases} 0\,, & \text{if } \|\theta\| < M_\theta \ \text{ or } \ \sigma_{eq} < 0 \\ \sigma_{eq}\,, & \text{if } \|\theta\| \geq M_\theta \ \text{ and } \ \sigma_{eq} \geq 0 \end{cases} \tag{4.14}$$

$$\sigma_{eq} = \frac{-\gamma\varepsilon\theta^T\omega}{\|\theta\|^2} \tag{4.15}$$

The eBMRAC equations are $e = y - y_m$, Eq. (4.2), (4.11)–(4.15) with ω as defined in Section 2. The block diagram is shown in Fig. 4.3. The predicted error $\hat{e}$ state dynamics can be written as

$$\dot{\hat{x}}_e = A_c\, \hat{x}_e + k^* b_c\, [u - u_0], \qquad \hat{e} = h_c^T\, \hat{x}_e, \tag{4.16}$$

with $k^* = k_p/k_m$. This allows us to obtain the state dynamics for the prediction error

$$\dot{x}_\varepsilon = A_c\, x_\varepsilon + k^* b_c\, [u_0 - u^*], \qquad \varepsilon = h_c^T\, x_\varepsilon, \tag{4.17}$$

$$\varepsilon = k^* W_m(s)[u_0 - u^*]; \tag{4.18}$$

Note that the prediction error does not depend on the filtered input.

The following properties are guaranteed by the projection based adaptation law:

Theorem 5 *Consider the error system described by (4.1)-(4.2) and the auxiliary errors (4.12),(4.17). The control signal is given by (4.11) with adaptation law (4.13)–(4.15). Assume that assumptions (A1)-(A4) hold, $\|\theta(0)\| \leq M_\theta$ and k_p is known. If*

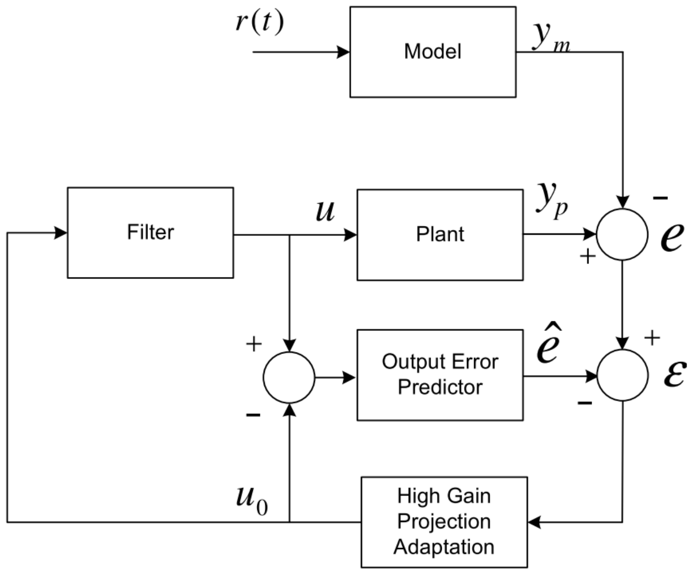

Figure 4.3: Extended BMRAC (eBMRAC) block diagram

τ is sufficiently small, then

i) $\|\theta\| \leq M_\theta, \forall t \geq 0$*;*

ii) $\|x_\varepsilon(t)\|^2 \leq c_1 e^{-\lambda_1 t} \|x_\varepsilon(0)\|^2 + \mathcal{O}(\gamma^{-1}), \forall t \geq 0$ *for some positive constants* c_1 *and* λ_1*;*

iii) *The prediction error* ε *tends asymptotically to zero;*

iv) e *tends exponentially to some small residual interval of order* τ*.*

Proof: see Appendix C

In the presence of unmodeled dynamics, it is expected that the prediction error tends to zero as gamma increases, since as γ tends to infinity, the resulting controller tends to the SSC. In this case, the eBMRAC should inherit the SSC properties of a free of chattering control signal and robustness to unmodeled dynamics.

The more general case, when only $sign(k_p)$ is known and unmodeled dynamics (including delays) are present, could also be considered following the same developments presented by [71]. It is worth of notice, however, that even though these results are not yet theoretically established, they are expected to be obtained without redesign of the controller. Simulations results show the behavior for aforementioned cases.

4.4.1 Adaptation Law with Projection with Smooth Saturation function for disturbance rejection

In [73], a Binary Adaptive Controller (referred to as *dual mode* therein) is employed with a smooth saturation term to provide robustness to disturbances. This idea is also useful for the eBMRAC, since unmodeled dynamics can be treated as disturbances.

The modified adaptive law with smooth saturation is

$$\hat{u} = \theta^T \omega - \bar{d} \frac{\varepsilon}{\|\varepsilon\| + \delta} \tag{4.19}$$

where $\bar{d}$ and δ are positive gains.

The smooth saturation is continuous with respect to its argument and is illustrated in Fig. 4.4 To obtain a similar result to [73], we introduce the following

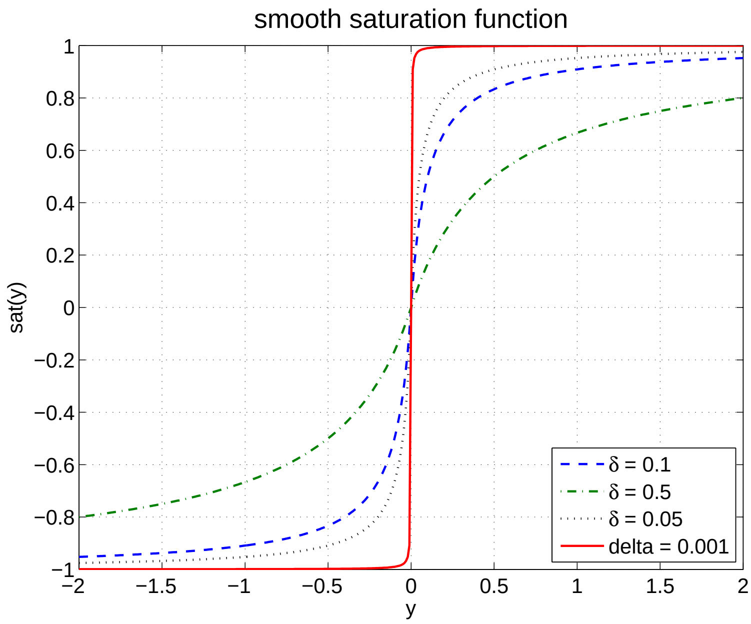

Figure 4.4: Smooth saturation function

function

$$f(\|\varepsilon\|) = \|\varepsilon\| \, d_{max} - \bar{d} \frac{\|\varepsilon\|^2}{\|\varepsilon\| + \delta} \tag{4.20}$$

such that $f(\|\varepsilon\|) \geq 0$ if $\bar{d} \leq d_{max}$. The maximum value of $f(\|\epsilon\|)$ is

$$f(\|\varepsilon\|_{max}) = \varepsilon d_{max} c(\bar{d}) \tag{4.21}$$

with

$$c(\bar{d}) = (-1 + d_s) - \frac{\bar{d}\,(-1 + d_s)^2}{d_{max} d_s} \tag{4.22}$$

and

$$d_s = \sqrt{\frac{\bar{d}}{\bar{d} - d_{max}}} \tag{4.23}$$

Using this new law along with the previous results it is possible to state the following theorem.

Theorem 6 *Consider the plant (1.21) with $M = 1$ and reference model (1.22) with uniformly bounded disturbances $\|d(t)\| \leq d_{max}$ and the error system described by (4.1)-(4.2) and the auxiliary errors (4.12),(4.17). The control signal is given by (4.19) with adaptation law (4.13)–(4.15). Assume that assumptions (A1)-(A4) hold, $\|\theta(0)\| \leq M_\theta$ and k_p is known and an upper bound $\bar{d} \geq d_0 > d_{max}$ is known. If τ is sufficiently small, then the prediction error converges exponentially to a residual set of*

$$\|\varepsilon\| \leq \mathcal{O}(1/\sqrt{\gamma}) + \mathcal{O}(\delta c(d_0)/\lambda_{min}(Q))$$

where Q is the solution to the Lyapunov equation $A_c^T P + P A_c = -Q$ with $P b_c = h_c^T$. Without disturbances, the prediction error exponentially converges to a residual set

$$\|\varepsilon\| \leq \mathcal{O}(1/\sqrt{\gamma})$$

and ultimately tends asymptotically to zero.

Proof: see Appendix C

Corollary 3 *If γ and δ are $\mathcal{O}(1/\sqrt{\gamma})$, then $\|\epsilon\| \to 0$ as $\gamma \to \infty$.*

This theorem shows that it is possible to reject uniformly bounded disturbances by adding a smooth saturation term in the control law, resulting in a more robust controller.

4.5 Simulation Results

4.5.1 Robustness properties of the eBMRAC

It is expected that the eBMRAC to inherit some of the SSC properties, since the BMRAC tends to a sliding mode controller as the gain is increased. This makes the proposed controller more interesting, since in this case it would present certain advantages in comparison to both the BMRAC and the SSC alone. This section shows a few properties through numerical simulations.

We consider a first order unstable plant

$$G(s) = \frac{1}{s-2} \tag{4.24}$$

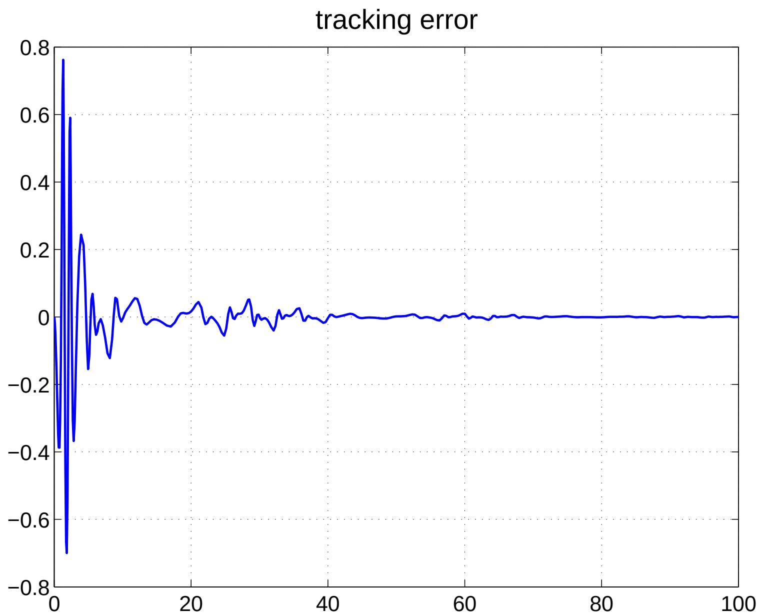

Figure 4.5: Tracking error with BMRAC

and the following reference model

$$W_m(s) = \frac{3}{s+3} \tag{4.25}$$

In this case, the ideal parameter vector and k^* are

$$\theta^* = [-5\ 3]\,; \qquad k^* = \frac{k_p}{k_m} = \frac{1}{3} \tag{4.26}$$

unless stated otherwise, we use $M_\theta = 6.5$.

Performance in the presence of unmodeled time-delays

It is shown in [71] that the SSC presents robustness to unmodeled time-delays in comparison to conventional VS-MRAC. This is a desirable property that is not verified for the BMRAC alone.

The results obtained for the previously presented plant with adaptation gain chosen as $\gamma = 10$ and reference signal $r(t) = sin(0.8t) + 0.5sin(t)$ are seen in Figs. 4.5-4.6. No time-delay is considered and it is possible to note the error convergence to zero. We now consider the same plant with time-delay, that is

$$G_{pd}(s) = \frac{1}{s-2}e^{-\mu s} \tag{4.27}$$

With $\mu = 0.1s$, simulation results obtained with the same choice of parameters as the previous case show that tracking performance is poor, as seen in Fig. 4.7-4.8.

Performance is fairly improved when the averaging filter and the prediction loop

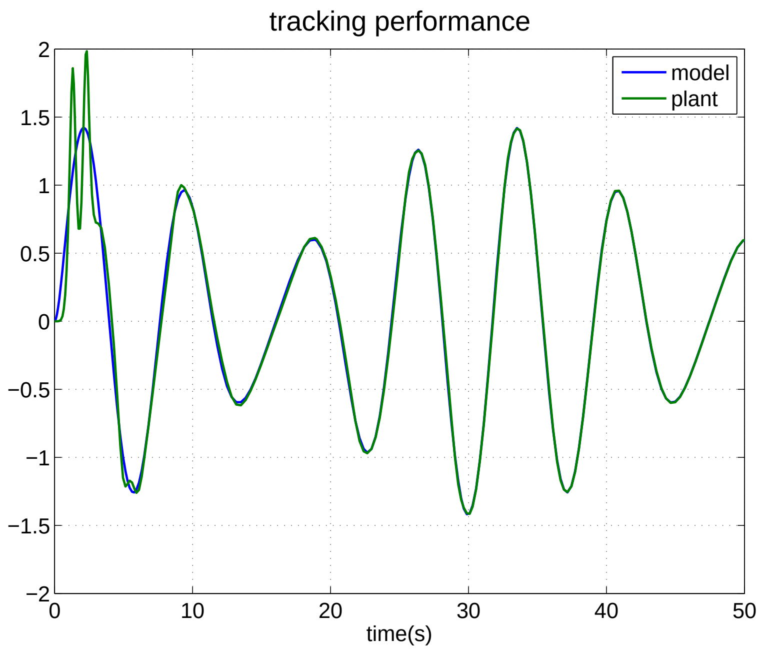

Figure 4.6: Tracking performance with BMRAC

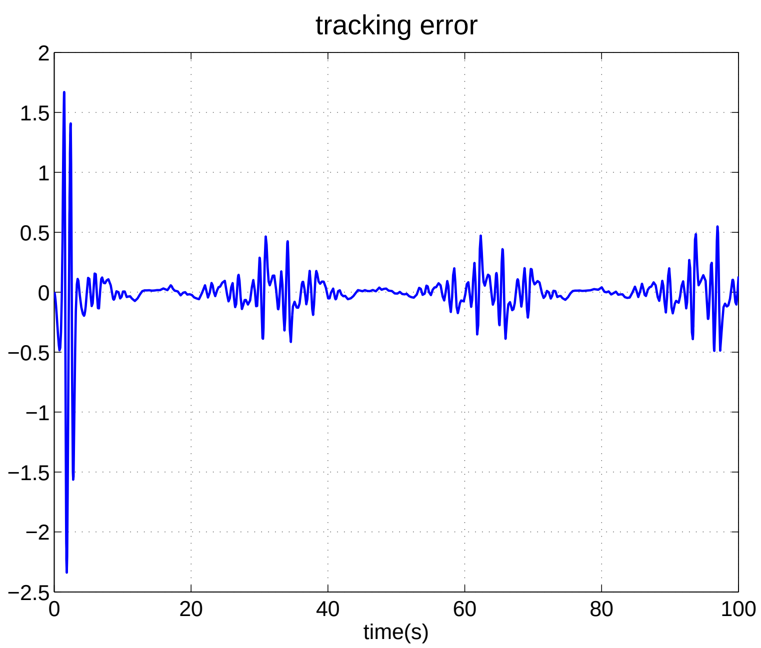

Figure 4.7: Tracking error with BMRAC in the presence of unmodeled time-delay

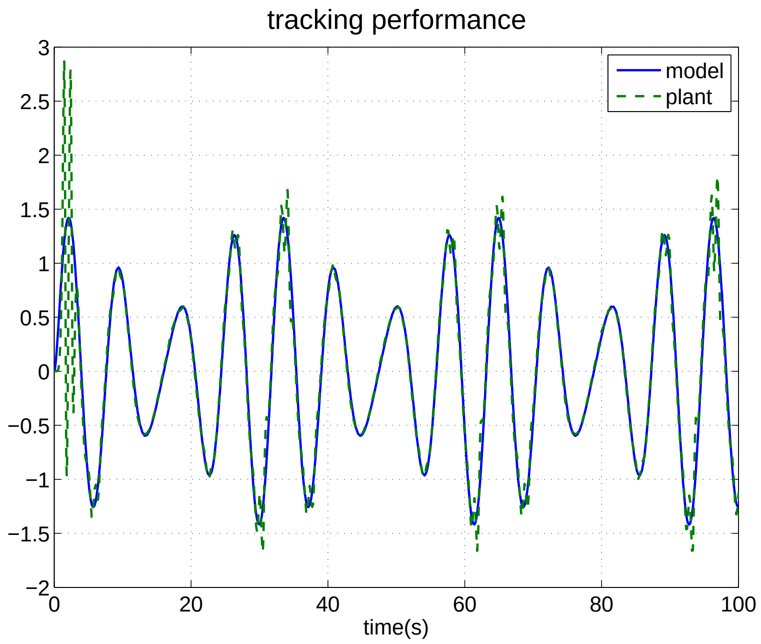

Figure 4.8: Tracking performance with BMRAC in the presence of unmodeled time-delay

are inserted. Even though convergence is not expected in the tracking error, as seen in Fig. 4.9, it can be made small with an appropriate choice of the averaging filter. In this case we use $\tau = 0.01s$ and set $k_{nom} = 1$.

Prediction error, on the other hand, converges to zero, as shown in Fig. 4.10. Tracking performance is seen in Fig. 4.11.

4.5.2 Performance in the presence of unmodeled dynamics

It is also interesting to verify the performance of eBMRAC for plants with unmodeled dynamics. Similarly to the case of systems with unmodeled time-delays, it can be shown that robustness properties are improved in comparison to BMRAC. We consider the following plant

$$G_{pd} = \frac{1}{s-2}\frac{1}{(\mu s+1)^2} \tag{4.28}$$

Reference signal is $r(t) = sin(0.8t) + 0.5sin(t)$. With $\mu = 0.05s$ and $\gamma = 100$, tracking is not achieved with BMRAC, as shown in Fig. 4.12.

Using the averaging filter it is possible to improve the performance. With $\tau = 0.03s$ and $k_{nom} = 1.5$, convergence is obtained for the prediction error, as seen in Fig.. 4.13. Since τ has to be tuned with a relatively high value, tracking performance is downgraded, as it is possible to note in Figs. 4.14-4.15. It is important to note, however, that decreasing τ does not improve tracking error properties in the presence

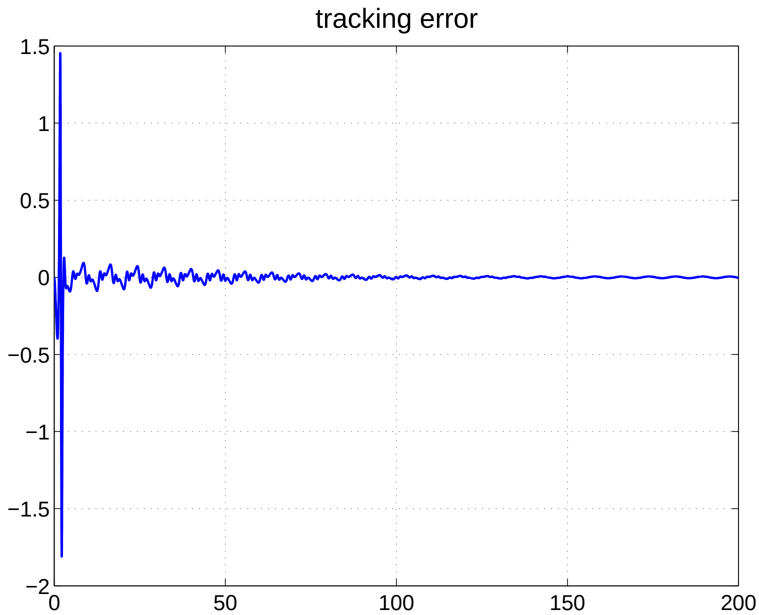

Figure 4.9: Tracking error with eBMRAC in the presence of unmodeled time-delay

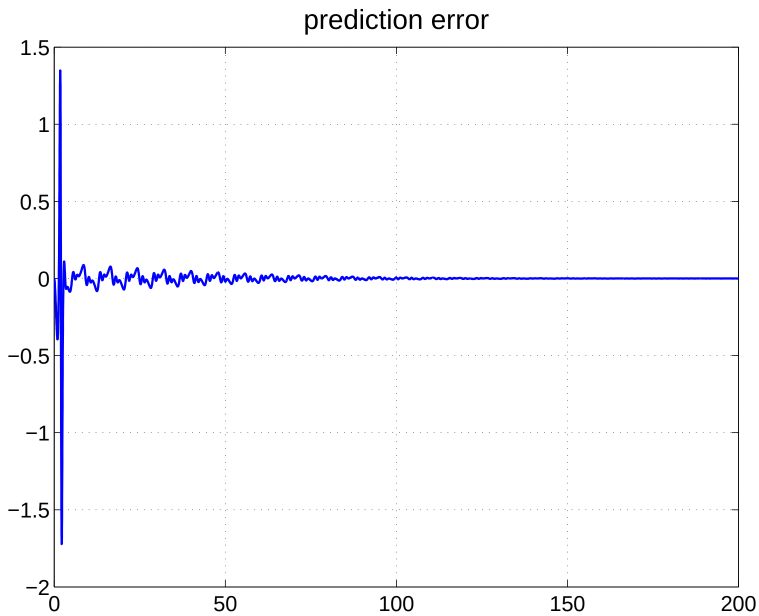

Figure 4.10: Prediction error with eBMRAC in the presence of unmodeled time-delay

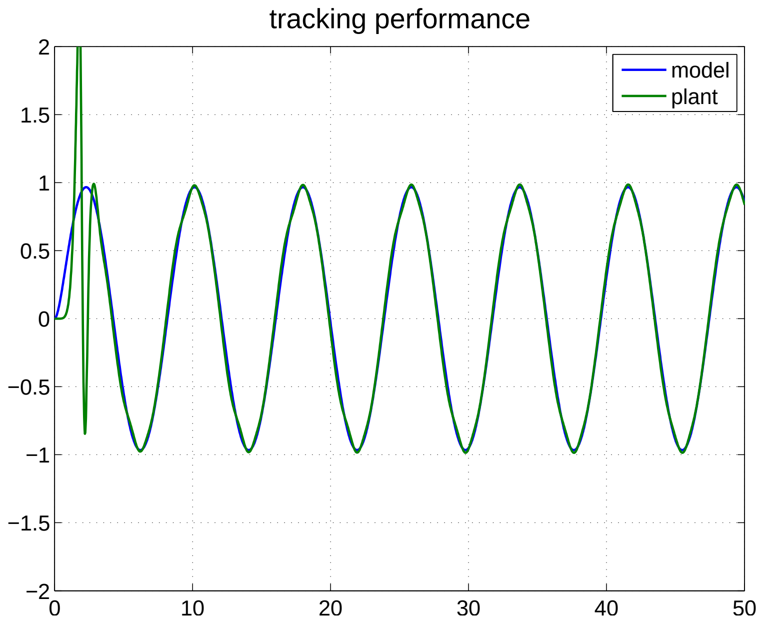

Figure 4.11: Tracking performance with eBMRAC in the presence of unmodeled time-delay

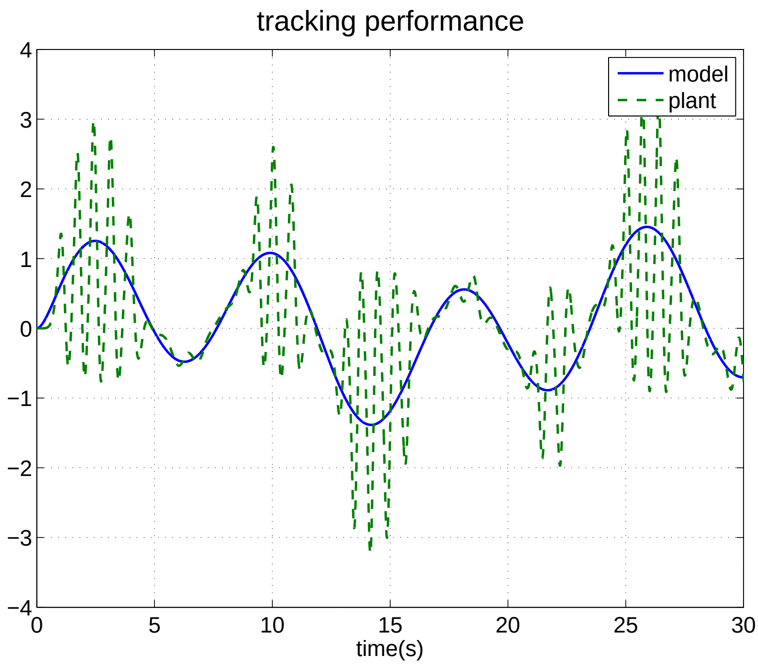

Figure 4.12: Tracking with BMRAC in the presence of unmodeled dynamics

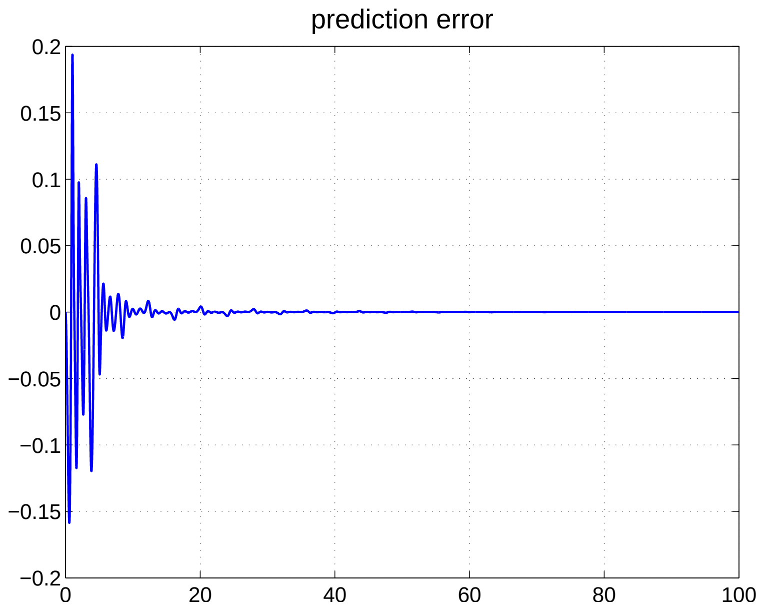

Figure 4.13: Prediction error with eBMRAC in the presence of unmodeled dynamics

of unmodeled dynamics. This can be understood intuitively, since as τ is decreased, also is the effect of the averaging filter, such that the controller approaches the regular BMRAC. Fig. 4.16 shows that the control is free of chattering.

Disturbance rejection using an augmented regressor

It is possible to reject constant perturbations through a simple modification of the regressor vector:

$$\omega_{aug} = [y_p(t) \quad r(t) \quad 1]$$

The following results are obtained for the first order plant of Eq. (4.27) with unmodeled time-delay of $\mu = 0.1s$ and an output disturbance of $d = 1$.

Fig. 4.17 shows the tracking error with a disturbance with the regular regressor. Performance is fairly improved with the augmented regressor as shown in Fig. 4.18. Similarly, prediction error does not converge with the regular regressor, as seen in Fig. 4.19. Convergence is obtained with the modification, see Fig. 4.20

4.5.3 Effect of k_{nom}

For the SSC in [71], the case where k_p is unknown can be dealt if an upper bound is known, such that k_{nom} is chosen to overestimate k^*. To the eBMRAC, this is still an open topic of research. However, simulations show that it is possible to obtain a stable behavior with k_p unkown without modifying the controller. The results with unmodeled dynamics is shown for different values of k_{nom}.

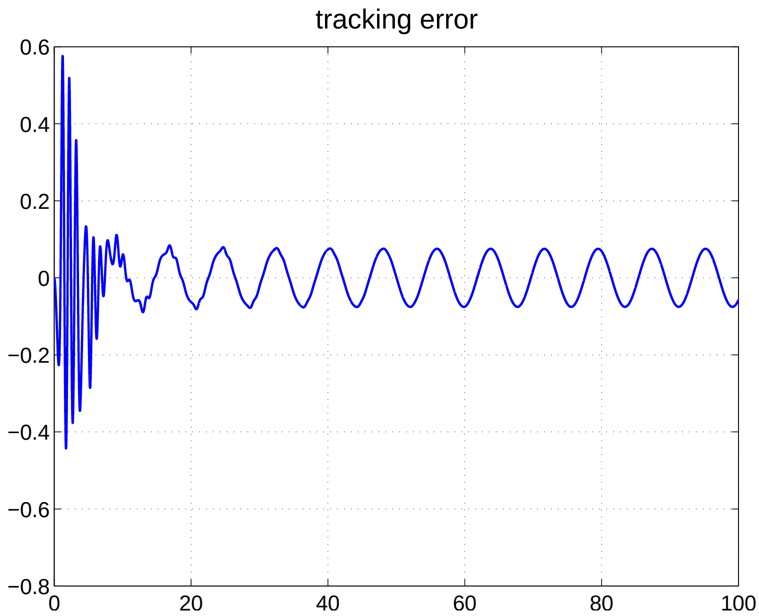

Figure 4.14: Tracking error with eBMRAC in the presence of unmodeled dynamics

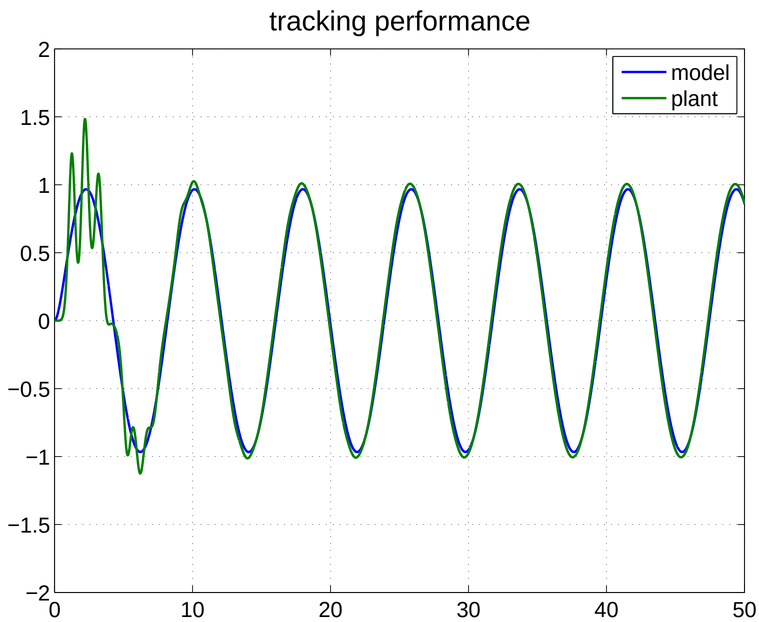

Figure 4.15: Tracking performance with eBMRAC in the presence of unmodeled dynamics

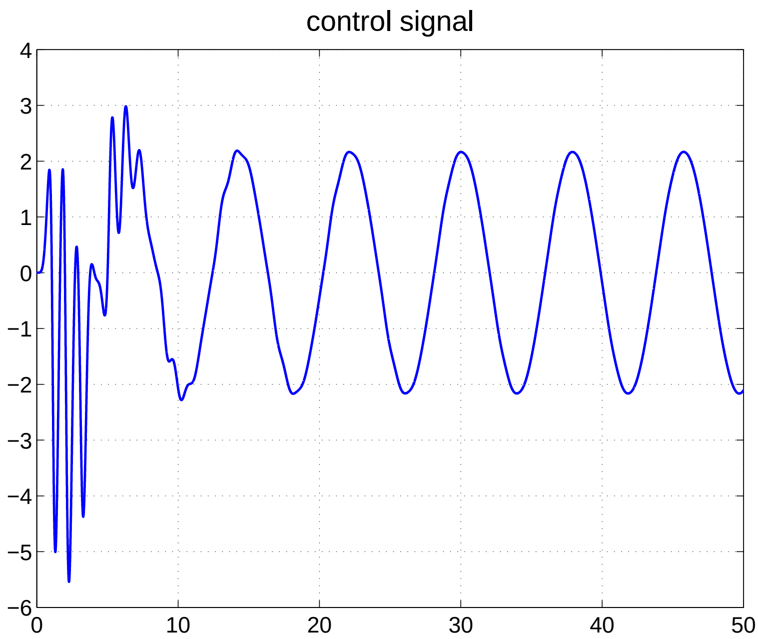

Figure 4.16: Control signal with eBMRAC in the presence of unmodeled dynamics

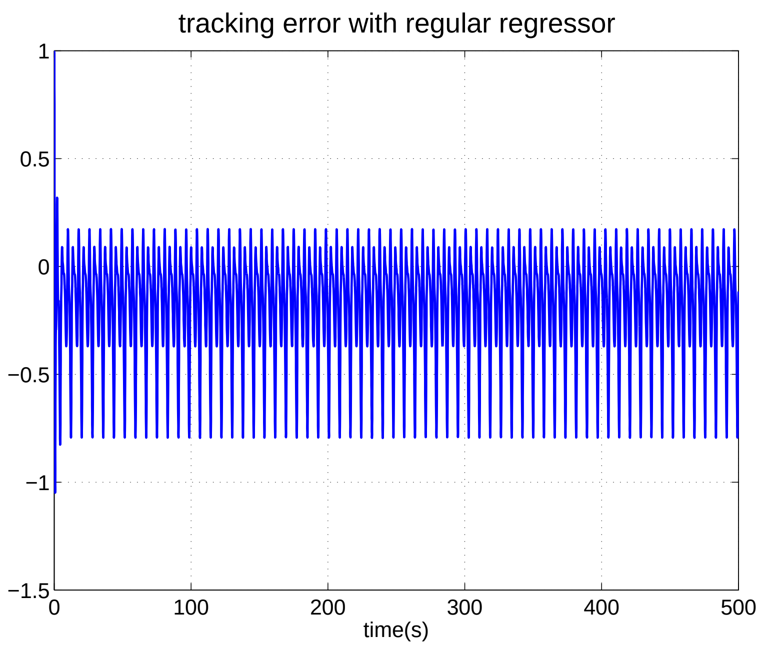

Figure 4.17: Tracking error with the regular regressor

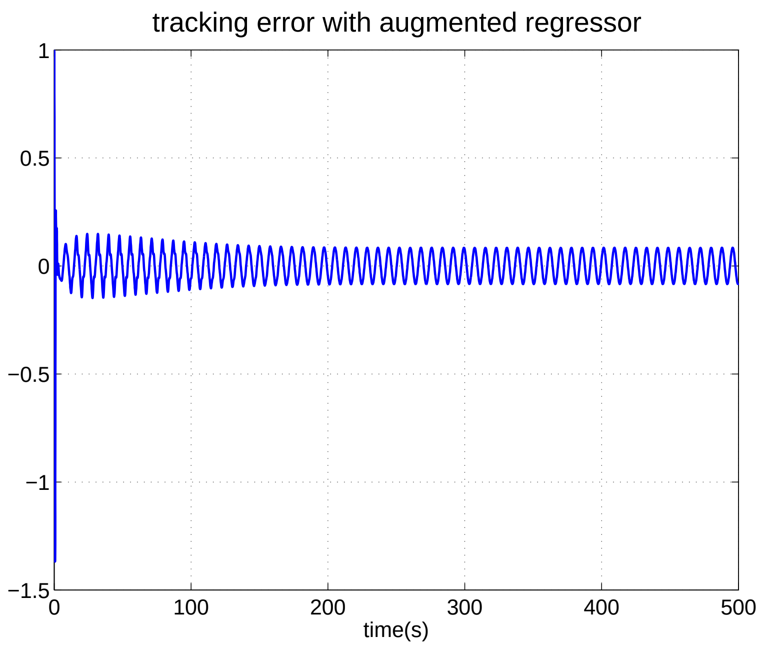

Figure 4.18: Tracking error with the augmented regressor

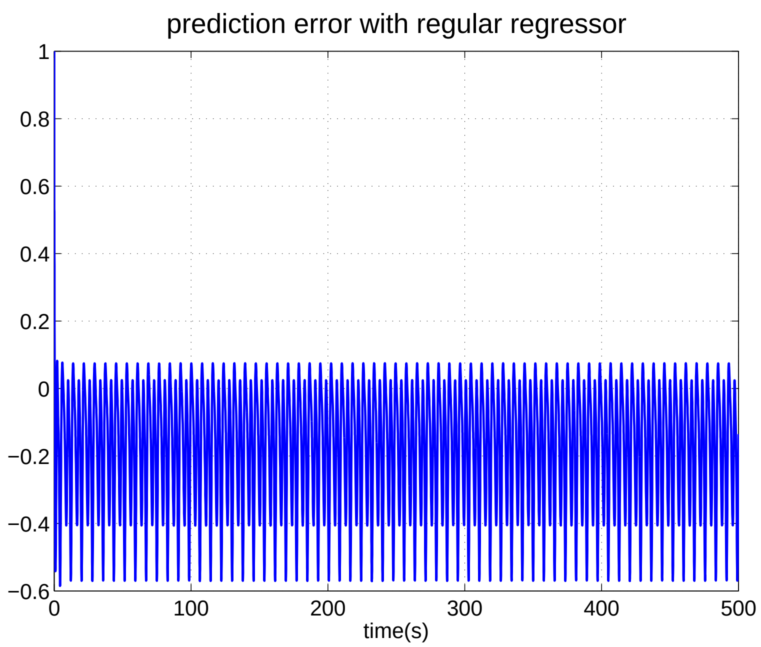

Figure 4.19: Prediction error with the regular regressor

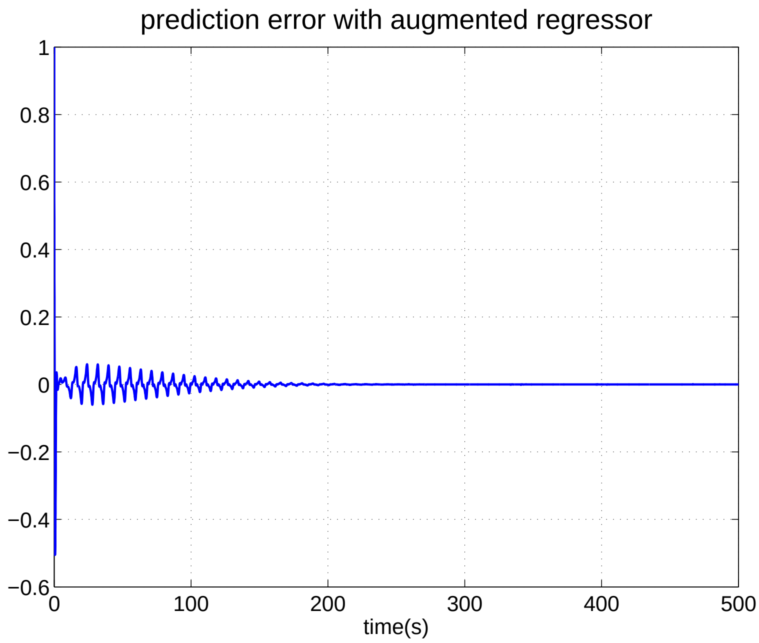

Figure 4.20: Prediction error with the augmented regressor

We consider the plant (4.28) with $\mu = 0.05s$ with $\tau = 0.03s$ and $\gamma = 10$, with the input $r(t) = sin(0.8t) + 0.5sin(t)$. Convergence is not obtained setting $k_{nom} = k^*$ as seen in Fig. 4.21.

If $k_{nom} = 2k^*$, convergence is obtained, as seen in Fig. 4.22–4.24. This can be explained by the predominance of a SPR loop formed by the plant and prediction error. This forces the prediction error to zero even in the presence of unmodeled dynamics if k_{nom} is sufficiently large. However, too large values may lead to larger tracking errors.

4.5.4 Transient and steady-state behavior properties

According to Theorem 5, the prediction error ε tends asymptotically to zero, but exponential convergence is only guaranteed to a residual set of order $\mathcal{O}(\gamma^{-1})$. Also, the tracking error tends exponentially to a residual set of order $\mathcal{O}(\tau)$. Simulations are used to illustrate these properties in the next subsections.

Effect of increasing gain in tracking and prediction errors

Comparison of transient and steady-state behavior of the eBMRAC for different values of adaptive gains is seen in Figs. 4.25–4.28, for $\gamma = 1,\ 5,\ 10$. In this example, the plant has the unmodeled delay given by Eq.(4.27) with $\mu = 0.05s$ and $\tau = 0.02s$. Initial plant output is set to $y = 4$. The reference signal is $r(t) = sin(0.8t) + 1/2sin(t)$.

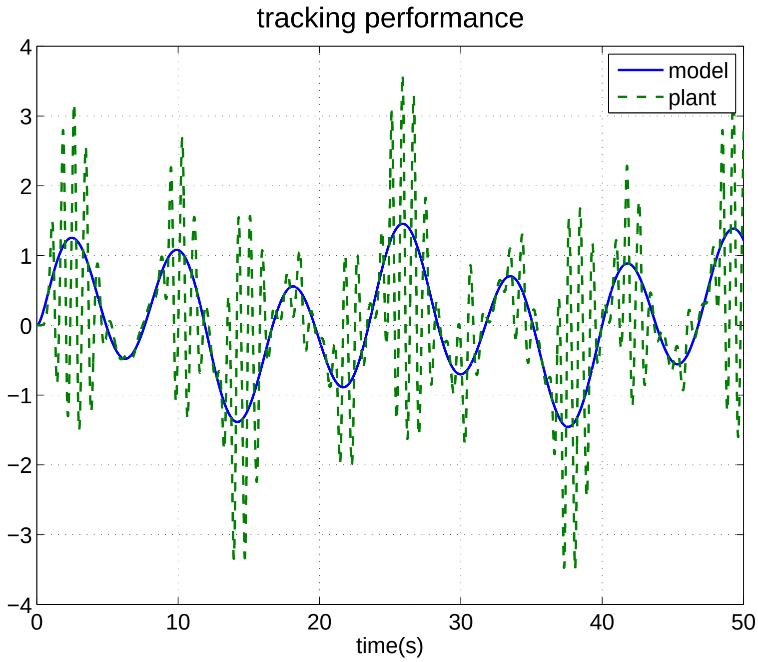

Figure 4.21: Tracking performance with unmodeled dynamics with $k_{nom} = k^*$

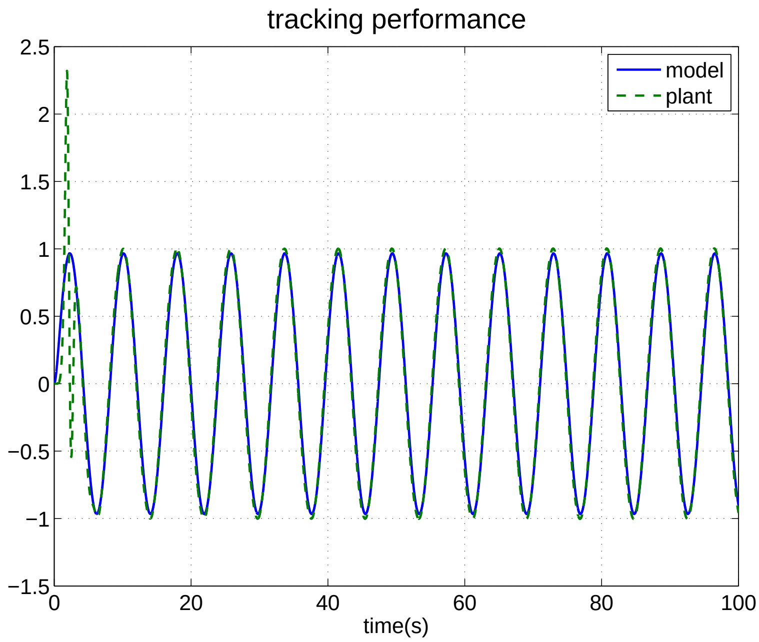

Figure 4.22: Tracking performance with unmodeled dynamics with $k_{nom} = 2k^*$

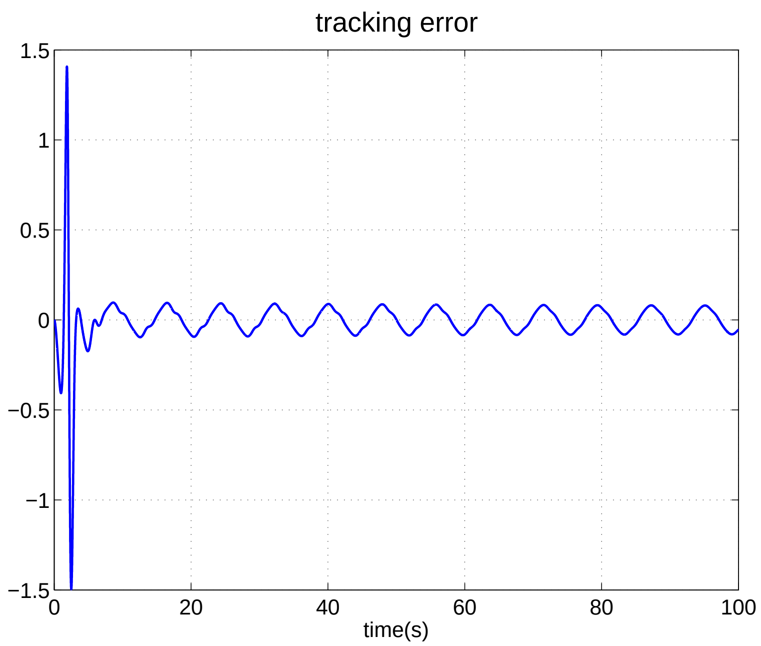

Figure 4.23: Tracking error with unmodeled dynamics with $k_{nom} = 2k^*$

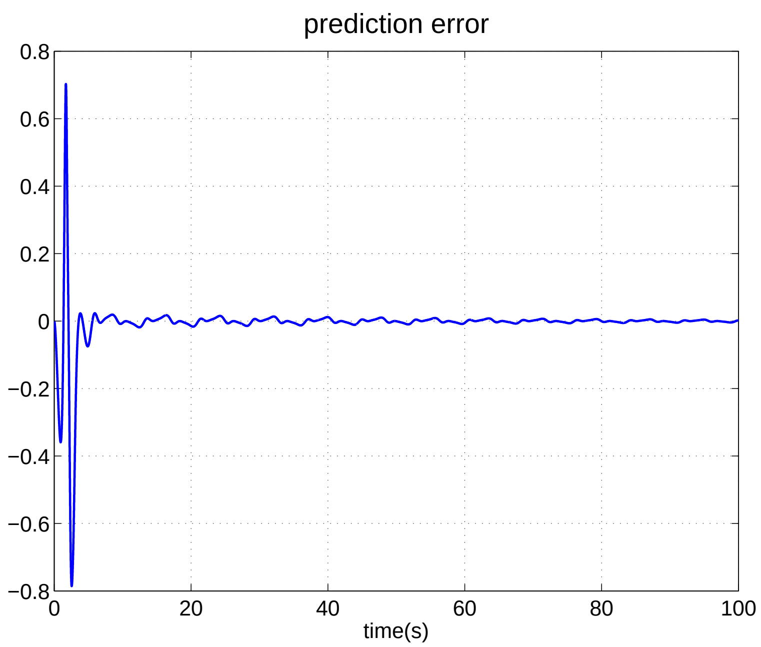

Figure 4.24: Prediction error with unmodeled dynamics with $k_{nom} = 2k^*$

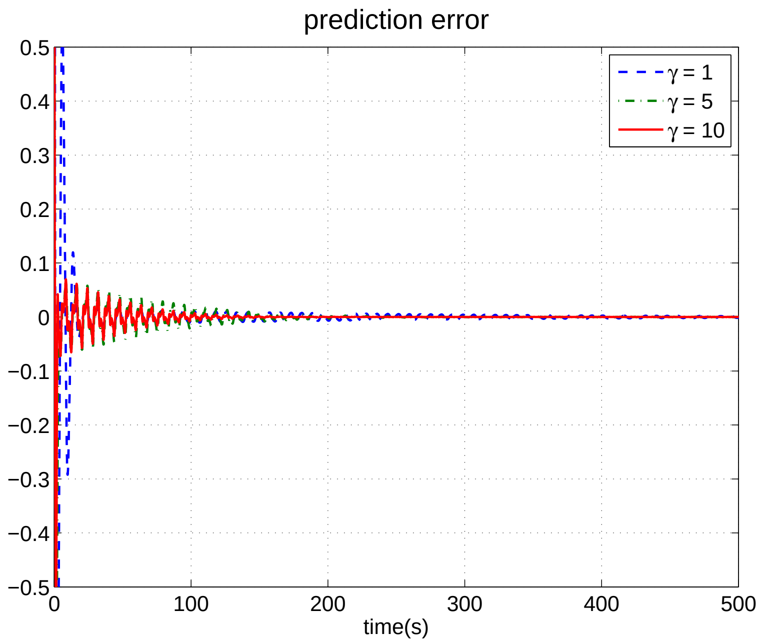

Figure 4.25: Prediction error for different values of γ

It is possible to note in Fig. 4.25 that the prediction error tends to zero asymptotically, as expected, irrespective of the value of the adaptation gain. Note, however, that convergence takes longer for lower values. It is also expected the exponential convergence to a residual set of order γ, which can be seen in Fig. 4.26.

Exponential convergence of the tracking error is also observed in Fig. 4.27 while Fig. 4.28 shows that it converges to a residual value that depends on τ irrespective of the adaptation gain.

It is also interesting to note that the control input is free of chattering, as seen in Fig 4.29

Effect of decreasing time constant of the averaging filter

The effect of the averaging filter can be further illustrated by Fig. 4.30. The averaging filter is switched among $\tau = 0.002s,\ 0.02s,\ 0.2s$ with $\gamma = 10$. Note that the tracking error decreases with τ. The figure indicates errors of amplitude or order 0.001, 0.01 and 0.1, respectively.

4.5.5 Second order unstable plant with time delay

Finally, a second order plant is used to illustrate the controller behavior. The following example is an unstable plant with relative degree one and time delay $\mu = 0.02s$.

$$G(s) = \frac{s+1}{(s+2)(s-2)} e^{-\mu s} \tag{4.29}$$

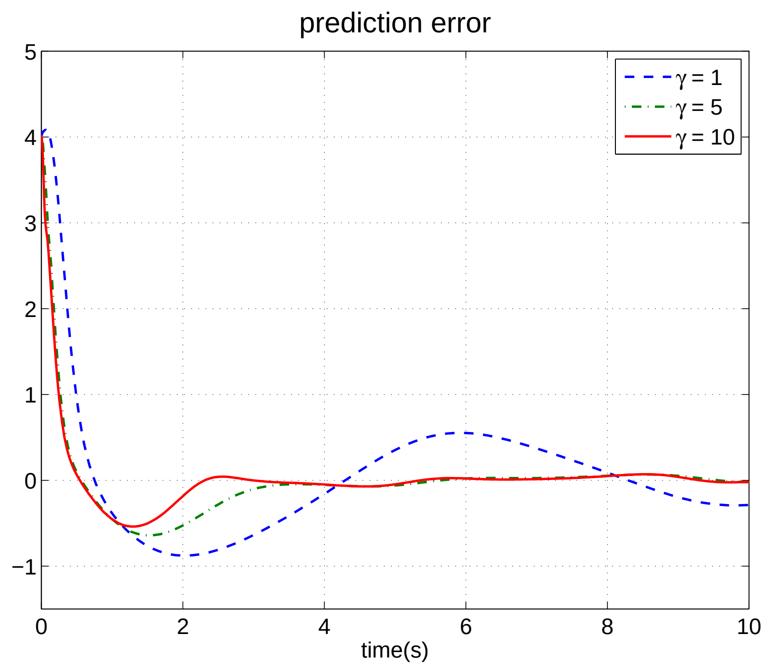

Figure 4.26: Detail of Fig. 4.25

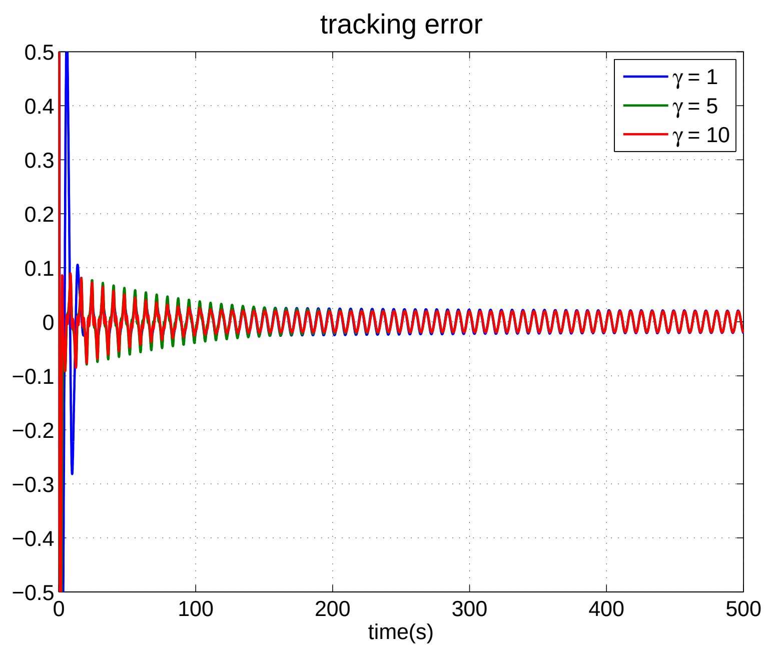

Figure 4.27: Tracking error for different values of γ

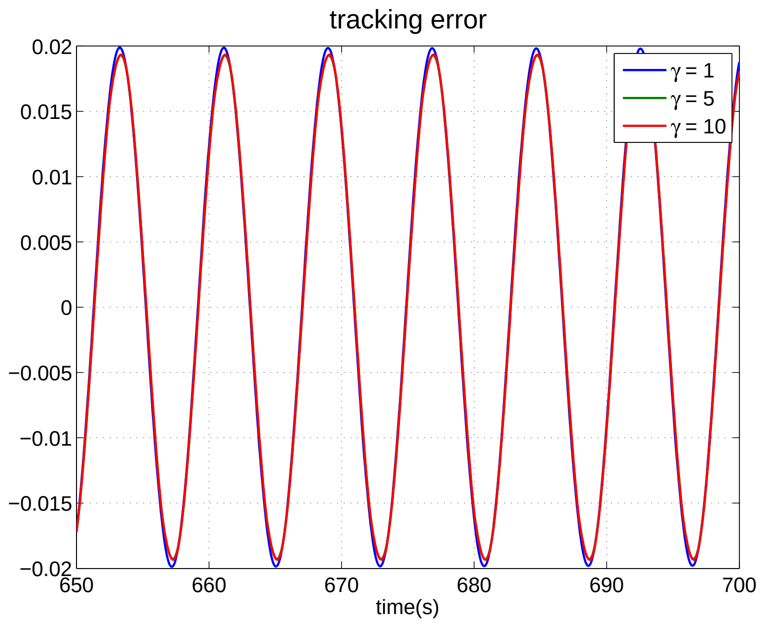

Figure 4.28: Steady state tracking errorfor different values of γ

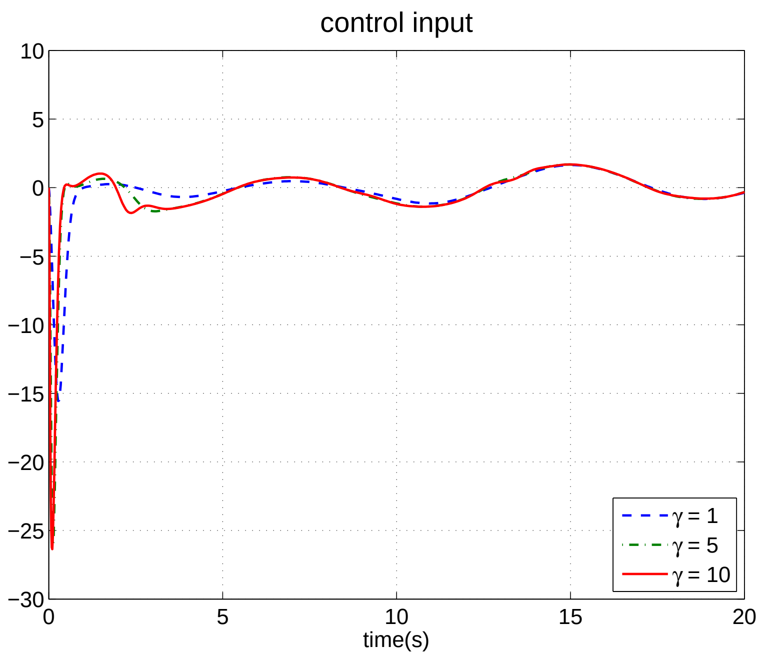

Figure 4.29: Control input with different values of γ

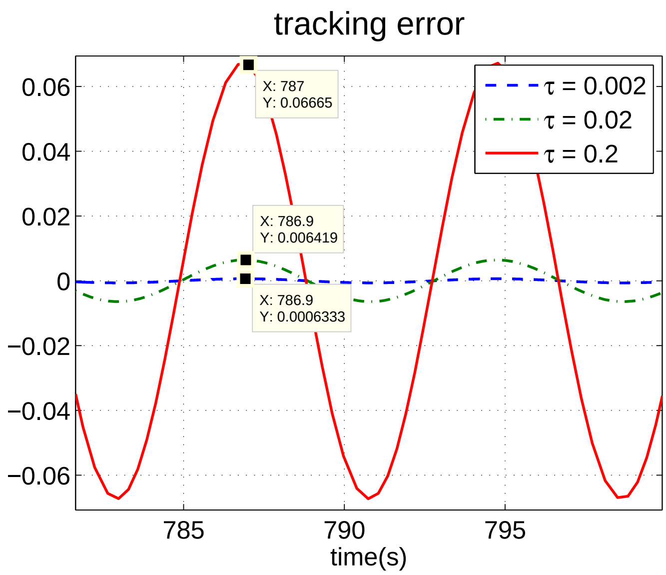

Figure 4.30: Tracking errors obtained for different values of τ showing residual convergence

The controller is desgined with $\gamma = 10$, $\tau = 0.01s$, $\Lambda(s) = 1/(s+1)$ and $k_{nom} = 0.5$. Reference input is $r(t) = sin(0.8)t$.

Convergence of prediction error is seen in Fig. 4.32, while tracking error converges to a residual set in Fig. 4.31. Tracking performance is seen in Fig. 4.33 and the control signal is free of chattering according to Fig. 4.34.

4.6 Limitations with L1-AC

Since the L1-AC presented in Section 1.5 shares common features with our proposed eBMRAC, we devote this section to a comparison of both architectures. The L1-AC achieves good regulation properties with guaranteed transient and robustness, but fails to track time varying references. Also, its architecture has to be modified to fit different applications. With our newly proposed technique, robustness and guaranteed transient behavior is also obtained and it is possible to overcome some of the L1-AC limitations. It is also interesting to note that in the L1-AC formulation, error states are uniformly bounded but initial conditions are assumed as zero. In the eBMRAC, tracking error is exponentially driven to a small residual set. Moreover, if the initial conditions are sufficiently small, trajectories would also stay in this set for all times.

Despite reports of successful applications as mentioned in Section 1.5, some recent work question the efficiency of the L1-AC such as [66] and [67]. Criticisms include the use of excessively high adaptation gains, the inability to track a time-varying reference and the coincidence of L1-AC control signal with a full state PI controller,

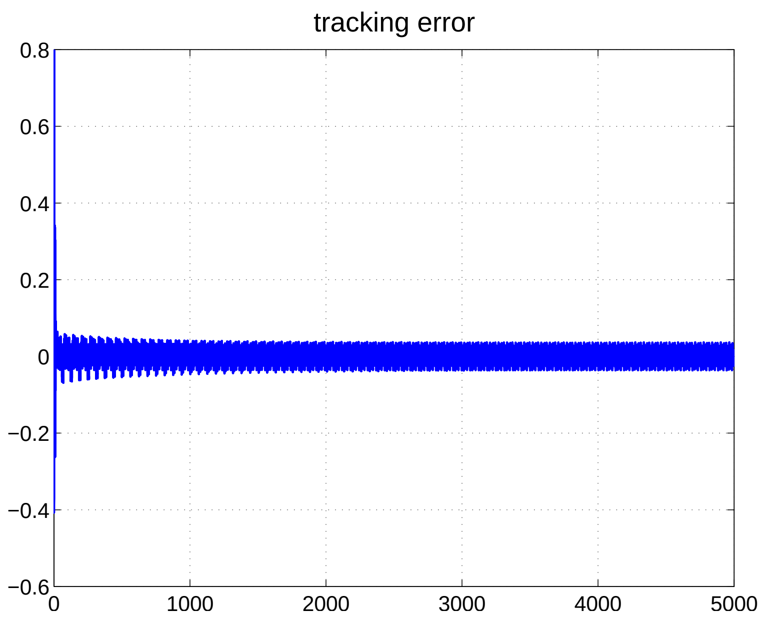

Figure 4.31: Tracking error for second order unstable plant

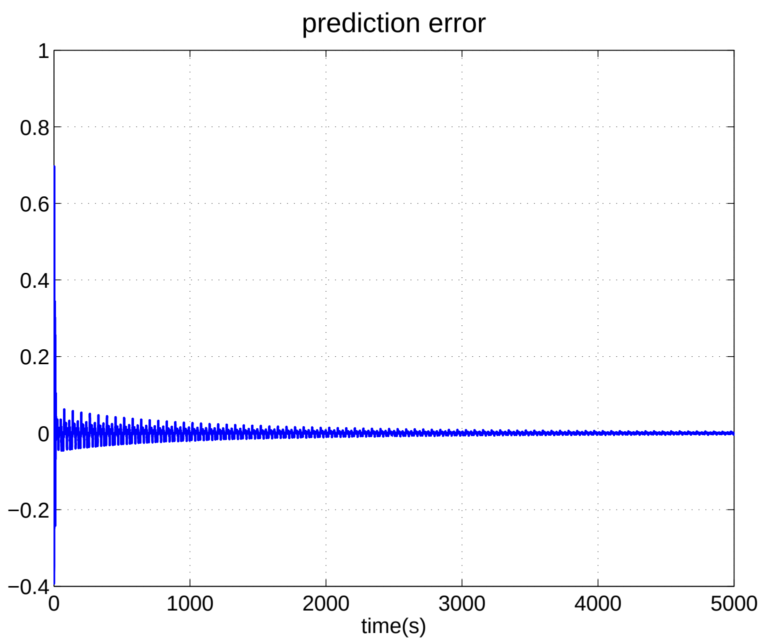

Figure 4.32: Prediction error for second order unstable plant

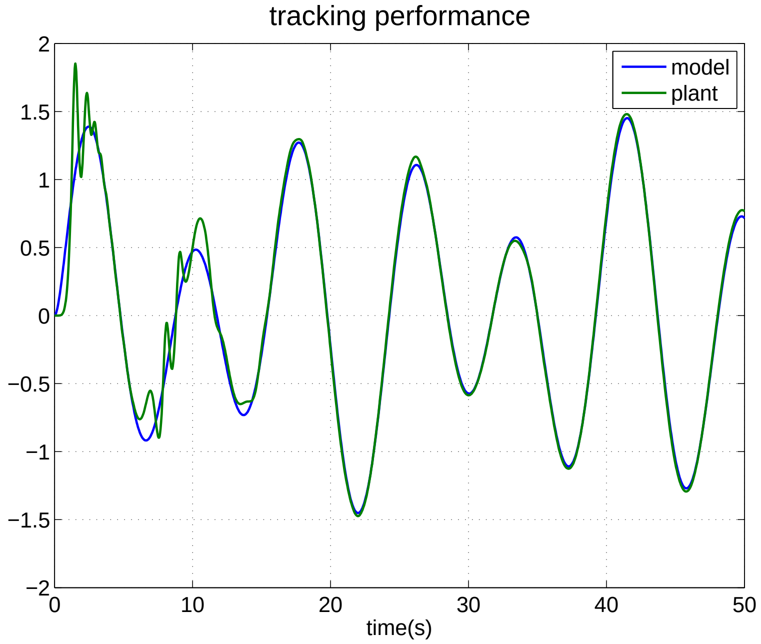

Figure 4.33: Tracking performance for second order unstable plant

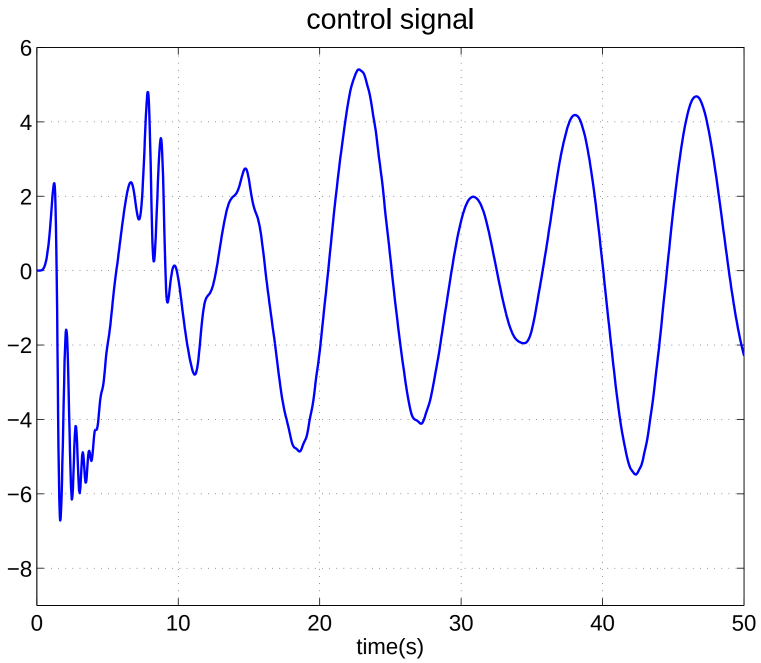

Figure 4.34: Control signal for second order unstable plant

which shows that adaptation is unnecessary in such scheme [74], [75]. It should also be noted that the L1-AC scheme is different according to the application. For instance, if the plant input gain is unknown, the algorithm has to be modified to a more complex architecture [58] (pp.35). This difficulty is similar for output feedback.

This can be illustrated using a first order stable LTI filter in L1-AC scheme, although it is also true for general filters according to [74]. Note that the filtered input can be rewritten as

$$\dot{u} = -k(u - \hat{\theta}^T x) \tag{4.30}$$

the control signal generated by (4.30) coincides with the output of a perturbed LTI PI controller

$$\dot{v} = kb^{\dagger} A_m x - \mu k \tilde{\theta}^T x; \quad u = v - kb^{\dagger} x \tag{4.31}$$

where $b^{\dagger}$ is the pseudo-inverse of b, given by $b^{\dagger} = (b^T b)^{-1} b^T$. This means that if the parameter error converges to zero, the obtained controller converges to an LTI controller that could be obtained without adaptation.

It is also important to note that L1-AC analysis does not guarantee zero tracking error for time-varying reference signals, as shown in [58]. The same applies to parameter error, such that only the prediction error is assured to be uniformly bounded.

Note that this controller requires knowledge of input gain. Even though the L1-AC theory is able to extend the idea shown above to contour this limitation, it is important to note that this is achieved by changing the control architecture.

The proposed controller is able to overcome these difficulties, since it does not require knowledge of input gain and is able to track a time-varying reference with residual error using output feedback without the need to modify the control scheme.

4.6.1 First Order Plant

In order to show the efficiency of the proposed controller in comparison with L1-AC, we use a simple first order example. Consider the following plant, state predictor and filter:

$$\dot{x} = 3x + u + \theta x; \quad \dot{\hat{x}} = -2\hat{x} + u; \quad C(s) = \frac{c}{s+c} \tag{4.32}$$

It is desired to track a sinusoidal reference signal, given by $r(t) = 10\sin(0.5t)$. High-gain is used as suggested by [58]. In this case, $\Gamma = 10^4$ and $c = 160$. The unknown parameter is assumed to be in the set $\theta = [-10,\ 10]$. For $\theta = -5$ the result is seen in Fig. 4.35, note that the system output does not track the reference input. The same plant, reference model and filter is used for the eBMRAC, that is

$$G(s) = \frac{1}{s - 3 - \theta}; \quad W_m(s) = \frac{1}{s+1}; \quad F_{av} = \frac{1}{\tau s + 1} \tag{4.33}$$

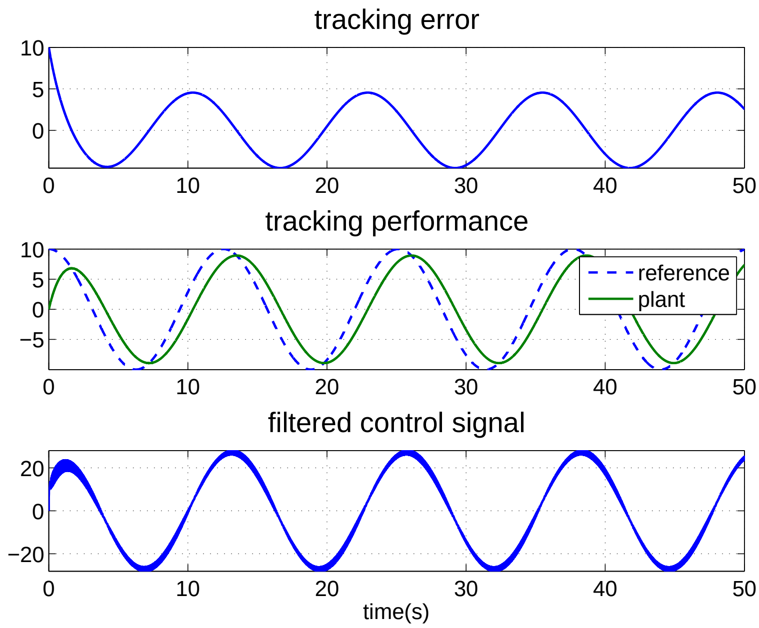

Figure 4.35: L1-AC performance for Example 1

Design parameters are chosen as $\gamma = 10$; $M_\theta = 10$ and $\tau = 0.02s$. The result is shown in Fig. 4.36, where it is possible to note a good tracking performance.

4.6.2 Second Order Plant

L1-AC:

To further compare the two schemes, the simulation results in this section consider the second order plant used in [33, 57], already including the unknown parameter θ of (1.75), that is:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & -1.4 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u; \qquad y = [1\ 0] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \tag{4.34}$$

The reference signal is $r(t) = 100cos(0.2t)$ and the filter is designed as $C(s) = 160/(s + 160)$. Adaptation gain is $\Gamma = 10^4$ and θ is assumed to be in the set $\theta_i = [-10,\ 10],\ i = 1, 2$.

The plant transfer function has relative degree $\rho = 2$. However, when dealing with state feedback, one can obtain an output of relative degree $\rho = 1$. The results of Figs. 4.37 reproduce the results shown in [57] and it is possible to note a poor tracking performance as well as oscillatory parameter values.

The limitations of L1-AC can be more clearly shown in two different scenarios: i) if the frequency is increased, which severely impairs the performance as seen in Fig. 4.38 when reference signal is $r(t) = 100cos(t)$; and ii) if the input gain is not known. In this case, the whole L1-AC scheme has to be redesigned, which is a

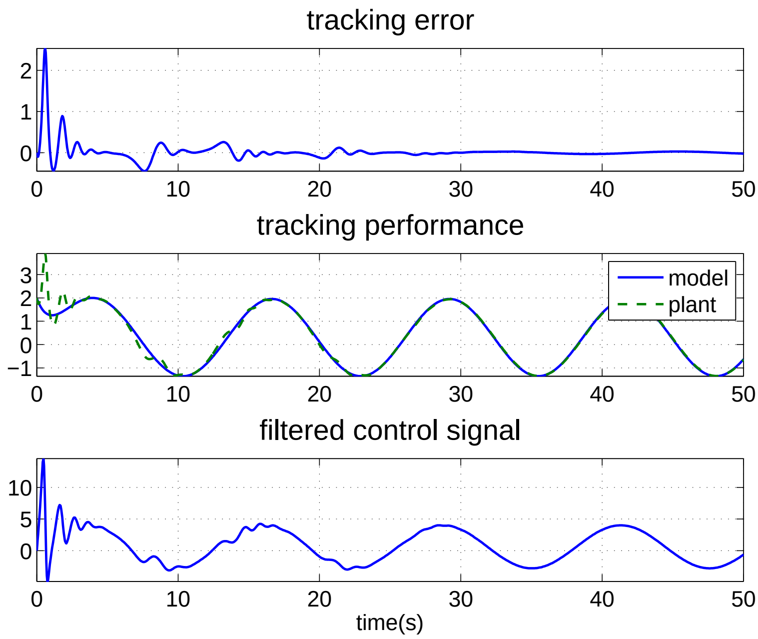

Figure 4.36: eBMRAC performance for for first order plant

restrictive constraint. Fig. 4.38 shows that it is not even possible to track a unit step input

eBMRAC:

The same plant of Eq. (4.34) is used assuming there is prior knowledge on the system states such that it is possible to obtain an output of relative degree $\rho = 1$. Considering both states are measurable, an output of relative degree one is obtained by combining the states such that the output is $\bar{y}_p = Pb_px_p$.

It is important to note that this is done for the sole purpose of providing a fair comparison, since the L1-AC is designed as state feedback. A more general version of the eBMRAC that would able to deal with systems of higher relative degree could be derived following the same steps as in [71]. For the sake of simplicity and for a more intuitive illustration of this comparison, we choose to deal with output feedback of a system with relative degree $\rho = 1$ in this example.

The averaging filter is designed with $\tau = 0.02s$, and I/O state variable filters are $\Lambda(s) = 1/(s+1)$. Projection adaptation parameters are $\gamma = 10$ and $M_\theta = 10$. Results show good tracking performance with residual error, as seen in Figs. 4.40, and a similar result when the frequency is increased, showing in Fig. 4.41. Zero tracking error is still obtained if k_p is unknown. The result when $k^{nom} = 1.2$ is shown in Fig. 4.42 Note that an excessively large gain is not needed and that the error can be made smaller choosing a smaller τ. It is important to stress that the eBMRAC inherits the robustness of the SSC to unmodeled dynamics such as delays

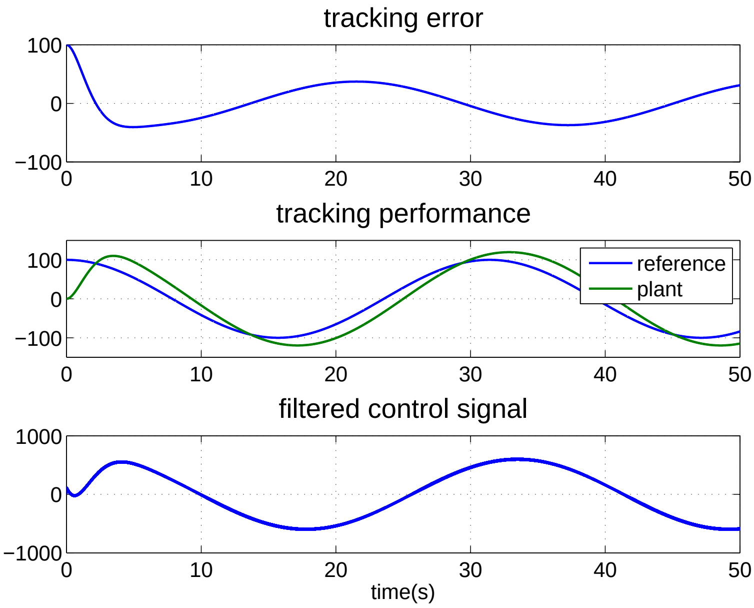

Figure 4.37: L1-AC: Second order plant with $r(t) = 100cos(0.2t)$

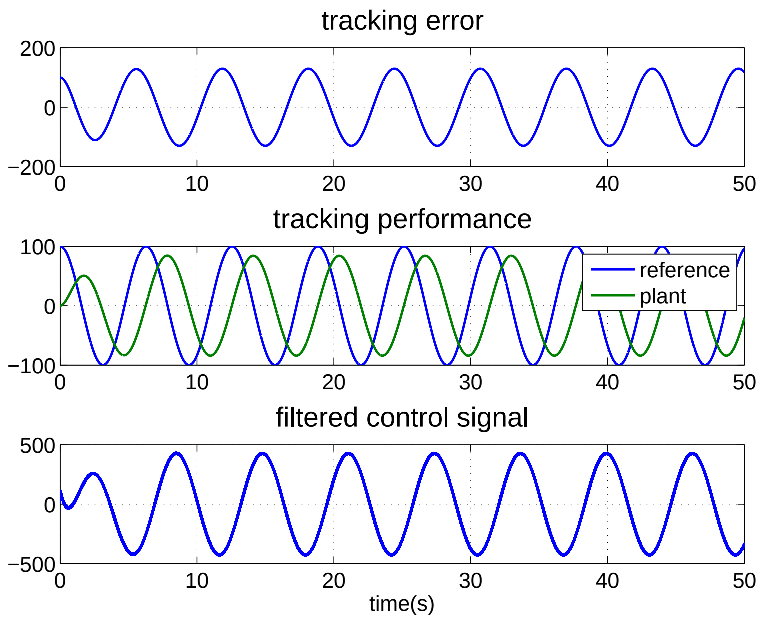

Figure 4.38: L1-AC: Second order plant with $r(t) = 100cos(t)$

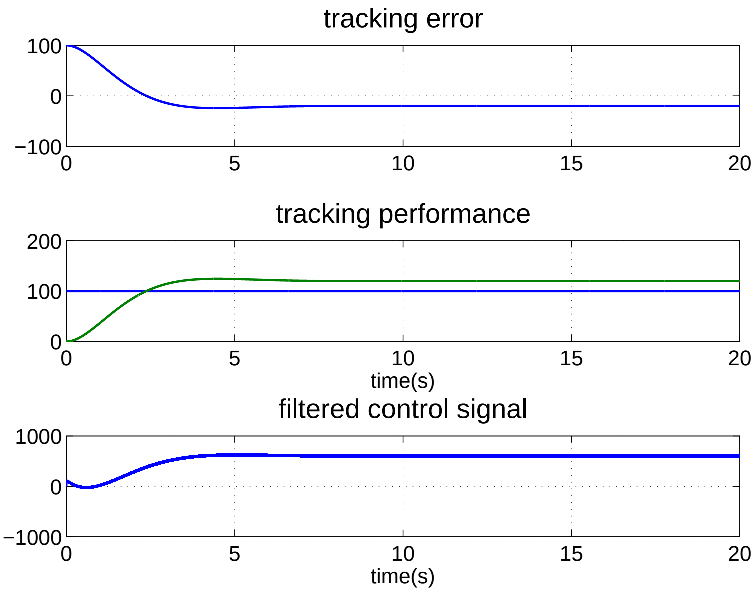

Figure 4.39: L1-AC step response for unknown input gain in Second order plant

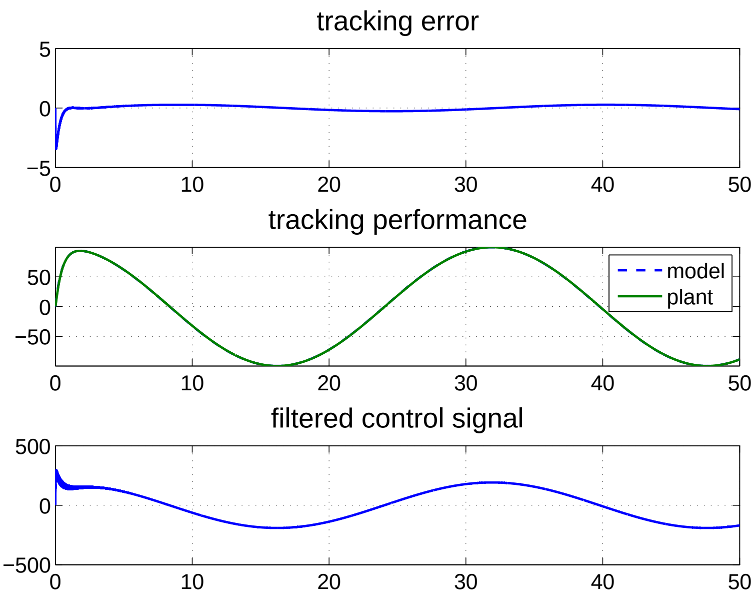

Figure 4.40: eBMRAC: Second order plant with $r(t) = 100cos(0.2t)$

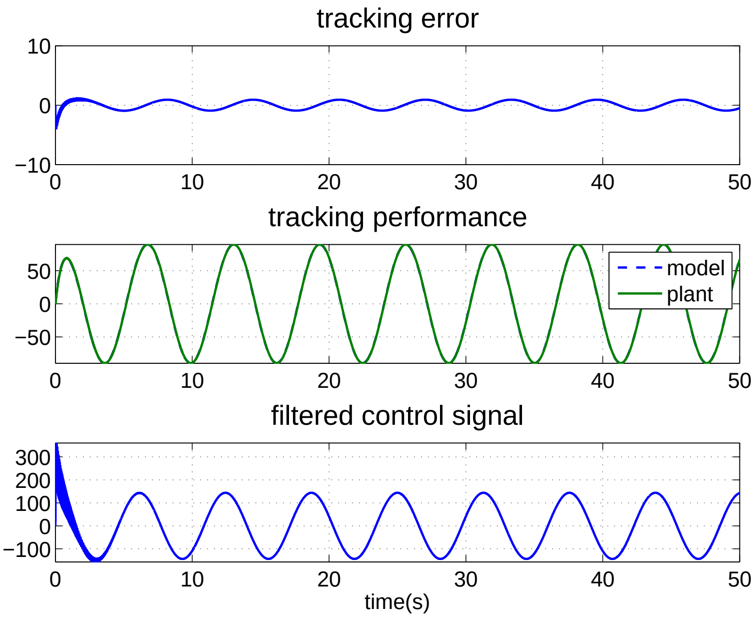

Figure 4.41: eBMRAC: Second order plant with $r(t) = 100cos(t)$

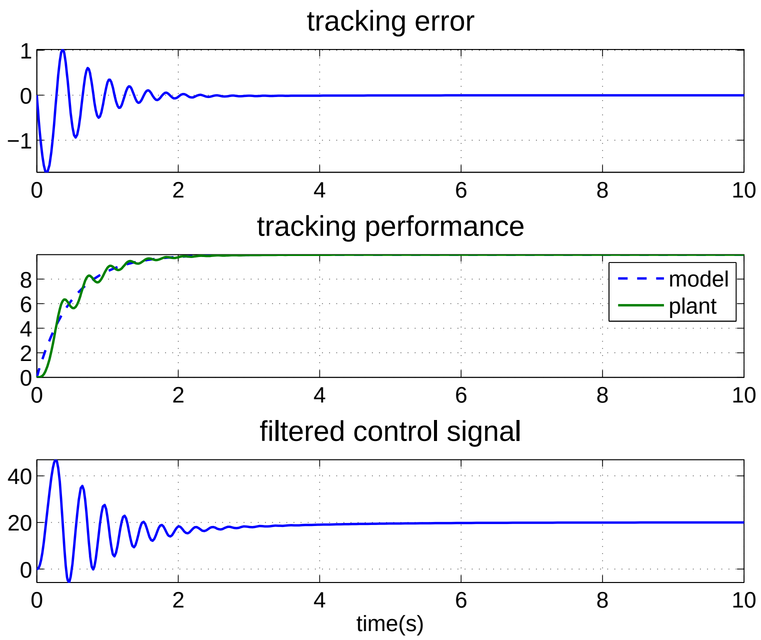

Figure 4.42: eBMRAC step response for unknown k_p for second order plant

and nonminimum phase dynamics.

4.7 Preliminary Results on the MIMO eBMRAC

The similarity with the BMRAC provides a bottom line to a MIMO version of the eBMRAC. We consider the same problem description as used for the BMRAC in Section 1.2 and use the idea to provide the MIMO counterpart to auxiliary errors. The error equation in input-output form is repeated below for convenience.

$$e = W_m(s)k_p\left[u - u^*\right] \tag{4.35}$$

The MIMO version of the predicted error can be chosen as

$$\hat{e} = W_m(s)K^{nom}[u - u_0] \tag{4.36}$$

where K_{nom} is an estimate of K_p.

If K_p is assumed known, as in the SISO case, the prediction error is similar and the result can be extended using similar steps.

$$\varepsilon = W_m(s)K_p[u_0 - u^*] \tag{4.37}$$

If we use $\bar{K}$ to account for uncertainties in K_{nom}, such as

$$K^{nom} = K_p\bar{K} \tag{4.38}$$

we obtain the following prediction error

$$\varepsilon = W_m(s)K_p\left[u_0 - \bar{u}\right] \tag{4.39}$$

where

$$\bar{u} = (I - \bar{K})u + \bar{K}u^* \tag{4.40}$$

It is conjectured that prediction error converges to zero if K_p os overestimated, however, tracking error increases with the estimative. This is an open problem in the present moment.

4.7.1 Simulation Results for the MIMO eBMRAC

We consider an unstable plant of uniform relative degree one

$$G_p(s) = \begin{bmatrix} \frac{1}{s-2} & 0 \\ 0 & \frac{1}{(s+1)} \end{bmatrix} K_p; \qquad K_p = \begin{bmatrix} 4 & 1 \\ 1 & 2 \end{bmatrix} \tag{4.41}$$

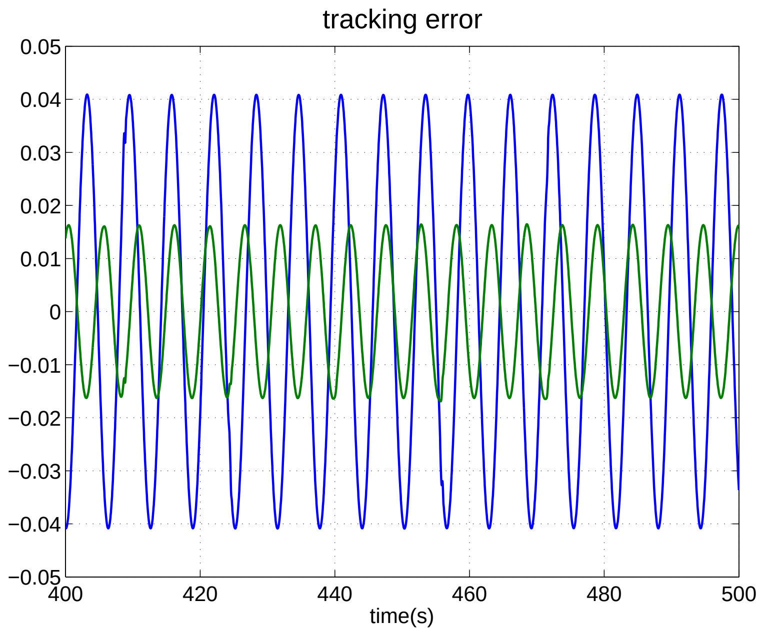

Figure 4.43: Tracking errors obtained for the MIMO example

Note that K_p is PDJ. We use the following reference model

$$W_m(s) = \begin{bmatrix} \frac{3}{s+3} & 0 \\ 0 & \frac{3}{s+3} \end{bmatrix} \tag{4.42}$$

And averaging filter

$$F_{av}(\tau s) = \begin{bmatrix} \frac{1}{\tau s+1} & 0 \\ 0 & \frac{1}{\tau s+1} \end{bmatrix} \tag{4.43}$$

Tracking errors converge to a residual set, as seen in Fig. 4.43. Tracking performance is shown in Fig. 4.45 and Fig. 4.44 shows that prediction errors converge to zero, as expected. The control signal is free of chattering, according to Fig. 4.46.

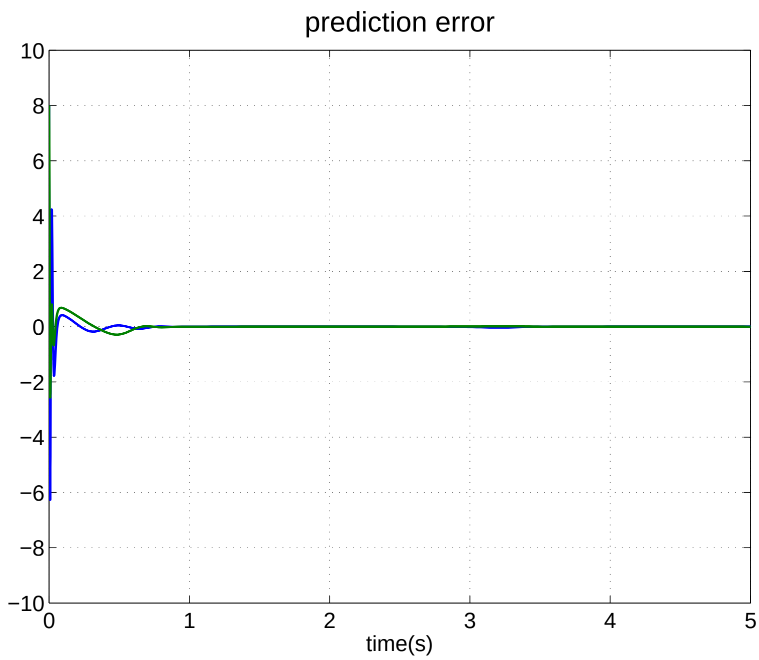

Figure 4.44: Prediction errors obtained for the MIMO example

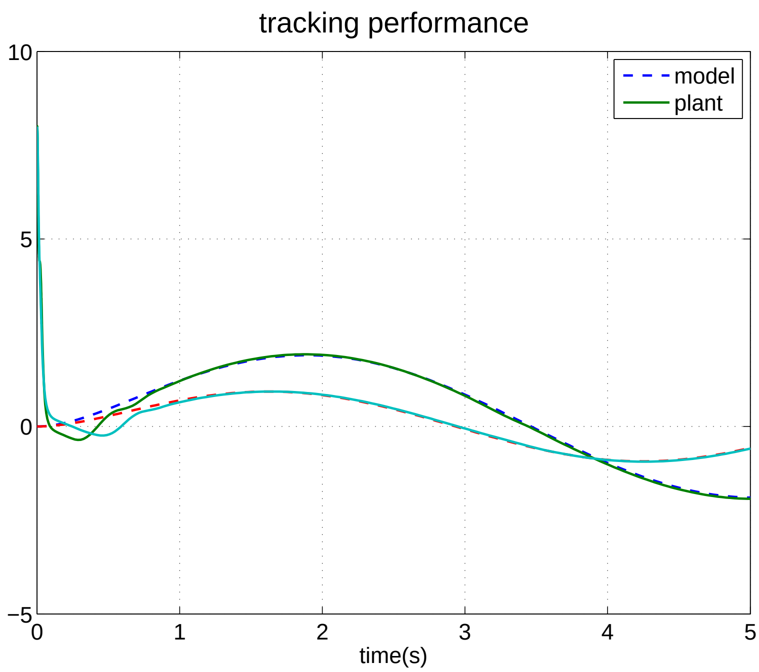

Figure 4.45: Tracking performance for the MIMO example

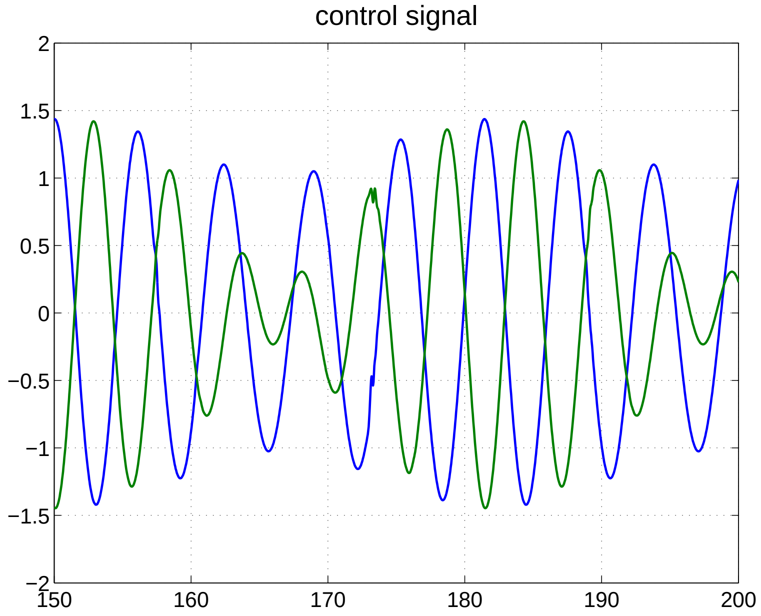

Figure 4.46: Control signals obtained for the MIMO example

Chapter 5

Conclusions and Future Work

This work proposed new solutions to Multivariable Model Reference Adaptive Control (MIMO MRAC) for plants of nonuniform arbitrary vector relative degree. Neither of these techniques are impaired by stringent assumption of symmetry or symmetrization of the plant high frequency gain (HFG). The proposed controllers are based on a PDJ condition related to the unknown HFG, which is a generic and less restrictive condition, particularly for uncertain systems. The augmented control parametrization of matrix factorization approaches is not necessary, making the new controller simpler.

A new technique based on the Bilinear error formulation was proposed considering both continuous-time and discrete-time cases. In contrast to conventional solutions, the new adaptive controller is simpler and we have also shown that, under certain circumstances, the symmetry condition can be relaxed even for the conventional design.

We also proposed the non-uniform arbitrary relative degree extension to the Multivariable Binary Model Reference Adaptive Control (BMRAC). Global exact output tracking for uncertain linear plants is obtained without requiring stringent symmetry assumptions on the High Frequency Gain and transient performance that outperforms standard MRAC techniques. To overcome the relative degree obstacle, we employed the multivariable version of the Global Robust Exact Differentiator (GRED) scheme, which achieves uniform global practical stability and exact tracking by switching a linear lead filter with a nonlinear one based on robust exact differentiators. The control signal is continuous and free of chattering.

A further extention to the BMRAC is proposed, namely the Extended BMRAC, obtained by a combination of the Smooth Sliding Control (SSC)and the BMRAC. It presents robustness and guaranteed transient behavior in the presence of unmodeled dynamics and time-delays, which is an advantage in comparison to standard BMRAC. It is based on high gain adaptation with projection, input filtering and the use of a prediction loop, which are features also used by the L1-AC controller. We show that the eBMRAC is able to overcome fundamental limitations of the L1-AC, such as poor tracking performance to time varying reference and the use of excessively large gain. Our new controller is simpler and its architecture does not need to be structurally redesigned in order to be applied to problems of different complexity.

Numerical simulations are presented to verify the analysis and show the effectiveness of the proposed techniques.

5.1 Future Work

5.1.1 Bilinear MRAC

The robustness issue is not yet addressed for this technnique, as well as its adaptation transient. The new algorithm is similar to conventional MRAC, so it is expected that robustness can be addressed using the same tools applied to the conventional case, such as projection adaptation laws or σ-modification.

5.1.2 GRED-BMRAC

Different architectures could be used to employ the GRED estimator in the BMRAC framework, such as estimating the error derivatives instead of the plant outputs. This is currently subject of ongoing work. The use of alternative differentiator topologies are worth of interest, such as the non-homogeneous version of the RED. The use of High Gain Observers is also a possible topic of future reserach

5.1.3 eBMRAC

The results obtained for the SISO case were only proved to relative degree one and known high frequency gain, such that these extensions are natural sequels to the present work. Also, a MIMO extension to this analysis is still an open research topic. Practical implementation of this algorithm is also an interesting issue.

5.1.4 General issues on MRAC

During the development of these techniques, it was possible to note how easily the number of parameters can grow in the conventional MRAC framework. For instance, a 4×4 with observability index $\nu = 2$ would need 64 parameters. This seems to be a complicating obstacle to quick convergence. To develop alternatives to the standard MRAC parametrization that would decrease the need of parameters is currently a challenging and interesting research topic.

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Appendix A

Robust Exact Differentiators (RED)

In the previous section, a BMRAC using a MIMO lead filter to estimate ξ_y was analyzed. From Theorem 3 the convergence of the error state is only guaranteed to a residual set.

Real-time differentiation of signals is an old and well-studied problem [69]. The output of an ideal differentiator is the exact derivative of any input signal, which is a considerably difficult task in practice since signals are corrupted by noise. High frequency components will result in high amplitude derivatives, corrupting the derivative of the signal of interest.

Even though robust estimates of derivatives can be obtained by a lead filter even in the presence of high frequency noise, for instance, these estimates are not exact. We are interested in a class of differentiators that are not only exact, but also able to reject small high frequency noise.

In [69],[76], a class of differentiators is proposed based on High Order Sliding Modes (HOSM). Considering an input signal $f(t)$, a function defined in $[0, \infty)$ consisted of an unknown base signal $f_0(t)$ whose nth-order derivative has a Lipschitz constant C_{n+1}, exact derivatives are obtained in finite time in the absence of noise. If the signal is corrupted by a bounded Lebesgue-measurable noise with unknown features, it is shown that asymptotically optimal performance is obtained.

In [76] it is shown that the best possible accuracy obtained by a differentiator to a i-th order derivative is proportional to

$$C_{n+1}^{i/(n+1)} \epsilon^{(n-i+1)/(n+1)}, \quad i = 0, \ldots, n$$

where C_{n+1} is the Lipschitz constant of the n-th order derivative and ϵ is the maximum noise magnitude.

The RED therefore solves the problem of obtaining robust derivatives in the presence of noise, and exact in the absence of noise. This is a useful tool to circumvent relative degree obstacles, such as done in [36]. In this work, we use the RED as a part of a scheme to obtain an arbitrary relative degree extension to the Binary-MRAC of [30].

A.1 First Order Differentiator

Consider the following auxiliary system

$$\dot{z} = v \tag{A.1}$$

and define the error variable $e = z - f_0$. The objective is to keep $e = 0$ in a second order sliding mode such that $e = \dot{e} = 0$. This implies that $z = f_0$ and $v = \dot{f}_0$. It is then possible to write

$$\dot{e} = -\dot{f}_0(t) + v, \quad \left|\ddot{f}_0(t)\right| \leq C_2 \forall t \tag{A.2}$$

The following Super-Twisting controller is then employed [76, 77]

$$\begin{cases} v = -\lambda_0 |e|^{1/2} sgn(e) + z_1 \\ \dot{z}_1 = -\lambda_1 sgn(e) \end{cases} \tag{A.3}$$

where λ_0 and λ_1 are positive constants. The first order differentiator presented in [76] is written in the following form:

$$\begin{cases} \dot{z} = v, \quad v = -\lambda_0 |z - f(t)|^{1/2} sgn(z - f(t)) + z_1 \\ \dot{z}_1 = -\lambda_1 sgn(z - f(t)) \end{cases} \tag{A.4}$$

where v and z_1 may be used as the output of the differentiator with $f(t)$ being the input signal. If the parameters λ_0 and λ_1 are properly chosen, the following equalities are established in finite time in the absence of noise:

$$z = f_0, \quad z_1 = v = \dot{f}_0 \tag{A.5}$$

A.2 Arbitrary Order Differentiators

The first order differentiator proposed in [76] was generalized to arbitrary order in [69] as follows

$$\begin{cases} \dot{\zeta}_0 = v_0, \\ v_0 = -\lambda_0|\zeta_0 - f(t)|^{n/(n+1)} sgn(\zeta_0 - f(t)) + \zeta_1 \\ \dot{\zeta}_1 = v_1, \\ v_1 = -\lambda_1|\zeta_1 - v_0|^{(n-1)/n} sgn(\zeta_1 - v_0) + \zeta_2 \\ \vdots \\ \dot{\zeta}_i = v_i, \\ v_i = -\lambda_i|\zeta_i - v_{i-1}|^{(n-i)/(n-i+1)} sgn(\zeta_i - v_{i-1}) + \zeta_{i+1} \\ \vdots \\ \dot{\zeta}_{n-1} = v_{n-1}, \\ v_{n-1} = -\lambda_{n-1}|\zeta_{n-1} - v_{n-2}|^{1/2} sgn(\zeta_{n-1} - v_{n-2}) + \zeta_n \\ \dot{\zeta}_n = -\lambda_n sgn(\zeta_n - v_{n-1}) \end{cases} \tag{A.6}$$

where $\lambda_1, \ldots, \lambda_n$ are positive constants and $f(t)$ is the input signal. It is possible to write the differentiator in a non-recursive form as follows:

$$\begin{cases} \dot{\zeta}_0 = -\kappa_0|\zeta_0 - f(t)|^{n/(n+1)} sgn(\zeta_0 - f(t)) + \zeta_1 \\ \dot{\zeta}_1 = -\kappa_1|\zeta_0 - f(t)|^{(n-1)/(n+1)} sgn(\zeta_0 - f(t)) + \zeta_2 \\ \vdots \\ \dot{\zeta}_i = -\kappa_i|\zeta_0 - f(t)|^{(n-i)/(n+1)} sgn(\zeta_0 - f(t)) + \zeta_{i+1} \\ \vdots \dot{\zeta}_i = -\kappa_n sgn(\zeta_0 - f(t)) \end{cases} \tag{A.7}$$

where the constants $\kappa_i, i = 0, \ldots, n$ are calculated from $\lambda_0, \ldots, \lambda_n$.

The following theorem, presented in [69], characterizes the convergence properties the RED.

Theorem 7 ([69]) *Consider system (3.27). Let the input signal $f_0(t)$ be a function defined on $[0, \infty)$ with the nth derivative having a known Lipschitz constant $C_{n+1} > 0$. If the parameters $\lambda_i,\ i = 0, \ldots, n$ are properly chosen, then in the absence of input noise the following equalities are true after a finite time transient process:*

$$\zeta_0 = f_0(t); \qquad \zeta_i = v_{i-1} = f_0^{(i)}(t),, \quad i = 1, \ldots, n$$

Proof: see [69]

The following result is also of great importance

Lemma 11 *Consider the system (A.6) and assume that the signals* $f(t), \dot{f}(t), \ldots, f^{(n)}(t)$ *are bounded. If* $|f^{(n+1)}(t)| \leq K_{n+1} \forall t$ *for some positive constant* K_{n+1}*, then the system state does not escape em finite time.*

Proof: see [68]. ■

Examples of RED tunning

In [69], guidelines to choose parameters λ_i are given. For a first order RED, considering $|\ddot{f}(t)| \leq C_2$, one has

$$\begin{cases} \dot{\zeta}_0 = v, \quad v = -1.5C_2^{1/2}|\zeta_0 - f_0(t)|^{1/2} sgn(\zeta_0 - f_0(t)) + \zeta_1 \\ \dot{\zeta}_1 = -1.1C_2 sgn(\zeta_1 - v_0) \end{cases} \tag{A.8}$$

Which is easily extended to a second order differentiator as follows, considering $|f^{(3)}(t)| \leq C_3$

$$\begin{cases} \dot{\zeta}_0 = v, \quad v = -3C_3^{1/3}|\zeta_0 - f_0(t)|^{2/3} sgn(\zeta_0 - f_0(t)) + \zeta_1 \\ \dot{\zeta}_2 = v, \quad v = -1.5C_3^{1/2}|\zeta_1 - v_0|^{1/2} sgn(\zeta_1 - v_0) + \zeta_2 \\ \dot{\zeta}_2 = -1.1C_3 sgn(\zeta_1 - v_0) \end{cases} \tag{A.9}$$

Appendix B

GRED-BMRAC proofs

In what follows, all k_i and κ_i are positive constants.

B.1 Proof of Theorem 3:

Firstly we show that $\|\vartheta(t)\| \leq M_\vartheta$ if $\|\vartheta(0)\| \leq M_\vartheta$ is bounded, such that $M_\vartheta > \|\vartheta^*\|$.

Consider the following Lyapunov function

$$2V_\vartheta = \vartheta^T\vartheta = \|\vartheta\|^2 \tag{B.1}$$

The derivative is

$$2\dot{V}_\vartheta = \dot{\vartheta}^T\vartheta + \vartheta^T\dot{\vartheta} \tag{B.2}$$

Recalling Eq. (3.34)

$$2\dot{V}_\vartheta = \left(-\vartheta\sigma - \gamma\Omega\bar{L}(\hat{\xi}_g - \xi_m)\right)^T \vartheta + \vartheta^T\left(-\vartheta\sigma - \gamma\Omega\bar{L}(\hat{\xi}_g - \xi_m)\right) \tag{B.3}$$

Which leads to

$$2\dot{V}_\vartheta = -2\sigma\vartheta^T\vartheta - \gamma\left(\Omega\bar{L}(\hat{\xi}_g - \xi_m)\right)^T \vartheta - \gamma\vartheta^T\left(\Omega\bar{L}(\hat{\xi}_g - \xi_m)\right) \tag{B.4}$$

Noting that $\left(\Omega\bar{L}(\hat{\xi}_g - \xi_m)\right)$ is scalar and recalling Eq. (3.36)

$$2\dot{V}_\vartheta = -2\sigma\|\vartheta\|^2 + 2\sigma_{eq}\vartheta^2 \tag{B.5}$$

$$\dot{V}_\vartheta = (\sigma_{eq} - \sigma)\|\vartheta\|^2 \tag{B.6}$$

and $(\sigma_{eq} - \sigma) \leq 0$ for $\|\vartheta\| \geq M_\vartheta$. Thus, the set $\|\vartheta\| \leq M_\vartheta$ is positively invariant and thus $\tilde{\vartheta}^T\tilde{\vartheta}$ is uniformely bounded by a constant.

Consider the following Lyapunov candidate, where P_1, P_2 and W_N are SPD,

$W_N = W \otimes I_N$ and $\tilde{\vartheta} = \vartheta - \vartheta^*$.

$$V = x_e^T P_1 x_e + \frac{1}{\gamma}\tilde{\vartheta}^T W_N \tilde{\vartheta} + x_\varepsilon^T P_2 x_\varepsilon \tag{B.7}$$

the derivative is

$$\begin{aligned} V &= x_e^T P_1 (A_c x_e + B_c K_p \Omega^T \vartheta) + (A_c x_e + B_c K_p \Omega^T \vartheta)^T P_1 x_e + \\ &+ \frac{1}{\gamma}\left[(-\vartheta\sigma - \gamma\Omega(\hat{\bar{e}}_L + \beta_\alpha))^T W_N \tilde{\vartheta} + \tilde{\vartheta}^T W_N (-\vartheta\sigma - \gamma\Omega(\hat{\bar{e}}_L + \beta_\alpha))\right] + \\ &+ x_\varepsilon^T P_2 \left(\frac{1}{\tau} A_\varepsilon x_\varepsilon + B_\varepsilon \dot{\xi}_y\right) + \left(\frac{1}{\tau} A_\varepsilon x_\varepsilon + B_\varepsilon \dot{\xi}_y\right)^T P_2 x_\varepsilon \end{aligned}$$

Since A_c and A_ε are Hurwitz and noting that $P_1 B_c K_p = (\bar{L} H_0)^T W$ and $\bar{e}_L = \bar{L} H_0 x_e$ it simplifies to

$$\begin{aligned} \dot{V} &= -x_e^T Q_1 x_e + 2(\hat{\bar{e}}_L)^T W \Omega^T \tilde{\vartheta} - \frac{1}{\tau} x_\varepsilon^T Q_2 x_\varepsilon - 2\frac{\sigma}{\gamma}\vartheta^T W_N \tilde{\vartheta} + \\ &- 2\tilde{\vartheta} W_N \Omega\left[\bar{L}(\xi_y + \varepsilon_l - \xi_m) + \beta_\alpha\right] + 2 x_\varepsilon^T P_2 B_\varepsilon \dot{\xi}_y \end{aligned} \tag{B.8}$$

Where we used the fact that $\hat{\bar{e}}_L = \bar{L}(\hat{\xi}_l - \xi_m)$ and $\xi_y = \hat{\xi}_l - \varepsilon_l$. From Eq. (3.15), and knowing that $W_N \Omega = \Omega W$ and $\bar{e}_L = \xi_y - \xi_m$ we have that

$$\begin{aligned} \dot{V} = -x_e^T Q_1 x_e - \frac{1}{\tau} x_\varepsilon^T Q_2 x_\varepsilon - 2\frac{\sigma}{\gamma}\vartheta^T W_N \tilde{\vartheta} - \tilde{\vartheta} W_N \Omega(\bar{L}\varepsilon_l + \beta_\alpha) + \\ +2 x_\varepsilon^T P_2 B_\varepsilon \bar{H} A_c X + 2 x_\varepsilon^T P_2 B_\varepsilon \bar{H} B_c K_p \Omega^T \tilde{\vartheta} + 2 x_\varepsilon^T P_2 B_\varepsilon \bar{H} B_c r \end{aligned}$$

Knowing that $\Omega^T \tilde{\vartheta} = \tilde{\theta}^T \omega$, $x_e = X - X_m$, $\omega_r = W_0(x_e + X_m)$ where $\omega_r = [\omega_u \ \omega_y \ y_p]^T$, $\theta_r^* = [\theta_1^* \ \theta_2^* \ \theta_3^*]^T$ and

$$W_0 = \begin{bmatrix} 0 & I & 0 \\ 0 & 0 & I \\ H_0 & 0 & 0 \end{bmatrix} \tag{B.9}$$

one has

$$\begin{aligned} \dot{V} = -x_e^T Q_1 x_e - \frac{1}{\tau} x_\varepsilon^T Q_2 x_\varepsilon - 2\frac{\sigma}{\gamma}\vartheta^T W_N \tilde{\vartheta} - (\bar{L}\varepsilon_l + \beta_\alpha)^T W \Omega^T \tilde{\vartheta} + \\ +2 x_\varepsilon^T P_2 B_\varepsilon \bar{H} A_c x_e + 2 x_\varepsilon^T P_2 B_\varepsilon \bar{H} A_c X_m + \\ +2 x_\varepsilon^T P_2 B_\varepsilon \bar{H} B_c K_p \tilde{\theta}_r^T W_0 x_e + 2 x_\varepsilon^T P_2 B_\varepsilon \bar{H} B_c K_p \tilde{\theta}_r^T W_0 X_m + \\ +2 x_\varepsilon^T P_2 B_\varepsilon \bar{H} B_c K_p \tilde{\theta}_4^T r + 2 x_\varepsilon^T P_2 B_\varepsilon \bar{H} B_c r \end{aligned}$$

Which can be simplified

$$\dot{V} = -x_e^T Q_1 x_e - \frac{1}{\tau} x_\varepsilon^T Q_2 x_\varepsilon - 2\frac{\sigma}{\gamma}\vartheta^T W_N \tilde{\vartheta} - (\bar{L}\varepsilon_l + \beta_\alpha)^T Q_6 [x_e + X_m] + \\ +(\bar{L}\varepsilon_l + \beta_\alpha)^T Q_7 r + 2x_\varepsilon^T [Q_3 x_e + Q_4 X_m + Q_5 r]$$

with

$$\begin{aligned} Q_3 &= P_2 B_\varepsilon \bar{H} B_c K_p \tilde{\theta}_r^T W_0 + P_2 B_\varepsilon \bar{H} A_c; \\ Q_4 &= P_2 B_\varepsilon \bar{H} B_c K_p \tilde{\theta}_r^T W_0 + P_2 B_\varepsilon \bar{H} A_c; \\ Q_5 &= P_2 B_\varepsilon \bar{H} B_c K_p \theta_4^T + P_2 B_\varepsilon \bar{H} B_c; \\ Q_6 &= W \tilde{\theta}^T W_0; \\ Q_7 &= W \theta_4^T \end{aligned}$$

Since θ is bounded and recalling that β_α is uniformly bounded by $\varepsilon_M = \tau K_R$, and $\|\varepsilon_l\| \leq \|x_\varepsilon\|$,

$$\begin{aligned} \dot{V} &\leq -k_1(Q_1)\|x_e\|^2 - \frac{k_2}{\tau}\|x_\varepsilon\|^2 - 2\frac{\sigma}{\gamma}\vartheta^T W_N \tilde{\vartheta} + k_3 \|x_\varepsilon\| \|x_e\| + \\ &+ k_4 \|x_\varepsilon\| + \tau k_5 \end{aligned}$$

Noting that from (3.10), the term $-2\frac{\sigma}{\gamma}\vartheta^T W_N \tilde{\vartheta}$ is non-positive since $\tilde{\vartheta} = \vartheta - \vartheta^*$ and $M_\vartheta \geq \|\vartheta^*\|$, we have

$$\dot{V} \leq -k_1 \|x_e\|^2 - \frac{k_2}{\tau}\|x_\varepsilon\|^2 + k_3 \|x_\varepsilon\| \|x_e\| + k_4 \|x_\varepsilon\| + \mathcal{O}(\tau)$$

We use the following equality to complete the squares:

$$-\left(\frac{\sqrt{k_2}}{\sqrt{2\tau}}\|x_\varepsilon\| - \frac{k_3\sqrt{2\tau}}{2\sqrt{k_2}}\|x_e\|\right)^2 = -\frac{k_2}{2\tau}\|x_\varepsilon\|^2 + k_3\|x_\varepsilon\|\|x_e\| - \frac{k_3^2}{2k_2}\tau\|x_e\|^2$$

which leads to

$$\dot{V} \leq -\left(\frac{\sqrt{k_2}}{\sqrt{2\tau}}\|x_\varepsilon\| - \frac{k_3\sqrt{2\tau}}{2\sqrt{k_2}}\|x_e\|\right)^2 - \left(k_1 - \frac{k_3^2}{2k_2}\tau\right)\|x_e\|^2 - \frac{k_2}{2\tau}\|x_\varepsilon\|^2 + k_4\|x_\varepsilon\|$$

and, consequently

$$\dot{V} \leq -\frac{k_1}{2}\|x_e\|^2 - \frac{k_2}{2\tau}\|x_\varepsilon\| - \left(\frac{k_1}{2} - \frac{k_3^2}{2k_2}\tau\right)\|x_e\|^2 + k_4\|x_\varepsilon\|$$

We proceed to complete the squares one more time using the following equation

$$-\left(\frac{\sqrt{k_2}}{2\sqrt{\tau}}\left\|x_\varepsilon\right\| - \frac{\sqrt{\tau}}{\sqrt{k_2}}k_4\right)^2 = -\frac{k_2}{4\tau}\left\|x_\varepsilon\right\|^2 + k_4\left\|x_\varepsilon\right\| - \tau k_5$$

And then we arrive at

$$\dot{V} \le \frac{k_1}{2}\left\|x_e\right\|^2 - \frac{k_2}{4\tau}\left\|x_\varepsilon\right\| - \left(\frac{k_1}{2} - \frac{k_3^2}{2k_2}\tau\right)\left\|x_e\right\|^2 + \mathcal{O}(\tau)$$

Assuming that $\tau \le \frac{k_1 k_2}{2k_3^2}$, it follows that

$$\dot{V} \le -\frac{k_1}{2}\left\|x_e\right\|^2 - \frac{k_2}{4\tau}\left\|x_\varepsilon\right\|^2 + \mathcal{O}(\tau)$$

Since $\|\vartheta\|$ is uniformly bounded, we have that

$$V \le [x_e\ x_\varepsilon]^T \begin{bmatrix} P_1 & 0 \\ 0 & P_2 \end{bmatrix} \begin{bmatrix} x_e \\ x_\varepsilon \end{bmatrix} + \mathcal{O}(\gamma^{-1}) \tag{B.10}$$

Such that we can write, with $z = [x_e\ x_\varepsilon]^T$

$$V - \mathcal{O}(\gamma^{-1}) \le z^T P z \le \lambda_{max}(P) z^T z \tag{B.11}$$

and

$$\dot{V} - \mathcal{O}(\tau) \le -z^T Q z \le -\lambda_{min}(Q) z^T z \tag{B.12}$$

$$\dot{V} \le -\lambda\left[V - \mathcal{O}(\gamma^{-1})\right] + \mathcal{O}(\tau) \tag{B.13}$$

with $\lambda = \lambda_{min}(Q)/\lambda_{max}(P)$. Using a comparison lemma it possible to show that there exist constants $c_z, a > 0$ such that $\|z(t)\| \le c_z e^{-a(t-t_0)}\|z(t_0)\| + \mathcal{O}(\sqrt{\tau}) + \mathcal{O}(\sqrt{1/\gamma})\ \forall z(t_0), \forall t \ge t_0 > 0$

B.2 Proof of Corollary 2:

From (3.6) and (3.15), one has $y_j^{(i)} = h_j^T A_c^{(i)} X,\ i = 1, \ldots, \rho_i - 1,\ j = 1, \ldots, m$. Since the augmented state X is uniformly bounded, then there exist positive constants, such that: $|y_j^{(i)}| \le K_i, \forall t \ge t_0 \ge 0,\ i = 0, \ldots, \rho_i - 1,\ j = 1, \ldots, M$. Moreover, the uniformly boundedness of $y_j^{(\rho_j)}, j = 1, \ldots, M$ follows from the fact that the augmented state $X(t)$, and the signals $u(t)$ and u^*, are uniformly bounded. The signals $y_j^{(\rho_j)}(t), j = 1, \ldots, M$ are given by:

$$y_j^{(\rho_j)} = h_j^T A_c^{\rho_j} X$$

Eq. (B.9) allows us to establish that $|w_r| \leq \kappa_1 |x_e| + \kappa_2$, since X_m, r and $\tilde{\theta}$ are bounded. Hence $y_j^{(\rho_j)}(t)$ can be upper bounded by

$$\left\| y_j^{(\rho_j)}(t) \right\| \leq \kappa_3 \|x_e(t)\| + \kappa_4$$

. Since $\|x_e(t)\| \leq R \; \forall t \geq T$, inequalities

$$\left\| y_{j_{[T,t]}}^{(\rho_j)} \right\|_\infty \leq C_{\rho_j}^{[j]}, \; j = 1, \ldots, M$$

can be established since the projection law ensures boundedness of $\tilde{\vartheta}$.

■

B.3 Proof of Theorem 4

The estimate given by the MIMO lead filter and the MIMO RED could be related to ξ_y in (3.15) as follows:

$$\hat{\xi}_l = \xi_y + \varepsilon_l, \qquad \hat{\xi}_r = \xi_y + \varepsilon_r\,, \tag{B.14}$$

where ε_l and ε_r are estimation errors. From (B.14), equation (3.32) can be rewritten as

$$\hat{\xi}_g = \xi_y + \varepsilon_g, \tag{B.15}$$

$$\varepsilon_g = \alpha(\tilde{\nu}_{rl})\varepsilon_l + [1 - \alpha(\tilde{\nu}_{rl})]\,\varepsilon_r\,. \tag{B.16}$$

The estimation error ε_g can be considered as an output disturbance. Thus, the GRED-BMRAC closed-loop error system (1.21), (3.1)–(3.3) can described by:

$$\dot{x}_e = A_c x_e + B_c K_p\,[u - \theta^{*T}\omega], \qquad e = H_0 x_e, \tag{B.17}$$

Note that $\{A_c, B_c, H_0\}$ is a nonminimal realization of

$$W_m(s) = \frac{K_m}{s + a_m},$$

and therefore is SPR. From (3.33), the estimation error $\varepsilon_g(t)$ can be rewritten as:

$$\varepsilon_g = \varepsilon_l + \beta_\alpha(\tilde{\nu}_{rl}(t))\,, \tag{B.18}$$

where by design $\beta_\alpha(\tilde{\nu}_{rl}(t))$ is uniformly bounded by

$$\|\beta_\alpha(\tilde{\nu}_{rl}(t))\| < \varepsilon_M, \text{ with } \varepsilon_M = \tau K_R\,.$$

Substituting (B.15),(B.18) into (3.34)–(3.36), it can be seen that GRED adaptive law is equivalent to lead adaptive law (3.24)–(3.26) with an output disturbance $\|\beta_\alpha(\tilde{\nu}_{rl}(t))\| \leq \varepsilon_M$.

Therefore, Theorem 3 holds if all signals of the GRED-BMRAC system belong to $L_{\infty e}$. In order to demonstrate that the condition is true, we only have to show that all signals in the MIMO RED system are $L_{\infty e}$. This property can be proved by contradiction. Suppose that the maximal interval of finiteness of the signals in the MIMO RED is $[0, T_M)$. During this interval, all conditions of Theorem 3 hold and thus all signals of the remaining subsystems of the GRED-BMRAC are bounded by a constant, and in particular $\left|y_j^{(i)}(t)\right|, i = 0, \ldots, \rho_j,\ j = 1, \ldots, M$, from Corollary 2. This leads to a contradiction with Lemma 10, whereby, the signals in the MIMO RED could not diverge unboundedly as $t \to T_M$. As a consequence of the continuation theorem for differential equations (in Filippov's theory), T_M must be ∞, which means that all signals are defined $\forall t \geq 0$. Thus, Theorem 3 is valid for the GRED-MRAC system and the closed-loop error system with state z is GEpS with respect to a residual set.

Now, we will analyze the ultimate convergence of the GRED-BMRAC. According to Corollary 1, for sufficiently small τ and sufficiently large γ the error state z is steered to an invariant compact set $D_R := \{z : |z(t)| < R\}$ in some finite time $T_1 \geq 0$. Consider the following Lyapunov candidate

$$V = x_\varepsilon^T P_2 x_\varepsilon \tag{B.19}$$

whose time derivative is

$$\dot{V} \;=\; -\frac{1}{\tau} x_\varepsilon^T Q_2 x_\varepsilon + 2 x_\varepsilon^T P_2 B_\varepsilon \dot{\xi}_y$$

following the previous steps

$$\dot{V} \;=\; -\frac{1}{\tau} x_\varepsilon^T Q_2 x_\varepsilon + 2 x_\varepsilon^T Q_3 x_e + 2 x_\varepsilon^T Q_4 X_m + 2 x_\varepsilon^T Q_5 r \tag{B.20}$$

$$\dot{V} \leq -\frac{k_2}{\tau} \|x_\varepsilon\|^2 + k_3 \|x_\varepsilon\| \|x_e\| + k_4 \|x_\varepsilon\| \tag{B.21}$$

Within D_R x_e can be upper bounded by $\|x_e\| \leq R$ such that

$$\dot{V} \leq -\frac{k_2}{\tau} \|x_\varepsilon\|^2 + \tau k_5 \tag{B.22}$$

Following similar steps to the proof of Theorem 1

$$\|x_\varepsilon(t)\| \leq c_\varepsilon e^{-a(t-t_0)} \|x_\varepsilon(t_0)\| + \tau k_6 \tag{B.23}$$

Since $\|\varepsilon_l\| \leq \|x_\varepsilon\|$, it is straightforward to show that for some finite $T_2 \geq T_1$, $\|\varepsilon_l\| \leq \bar{\varepsilon}_l$, where $\bar{\varepsilon}_l = \tau K_l$.

Since the MIMO RED is time invariant, its initial conditions can be considered to be at $t = T_1$. From Lemma 10 the initial conditions are finite. If the parameters λ_i^j are adjusted properly, then from Theorem 7 the estimation error $\varepsilon_r(t)$ converges to zero in some finite time $T_3 > T_1$.

Since K_R is chosen such that $\varepsilon_M > \bar{\varepsilon}_l + \Delta$ and from (3.33), it follows that after some finite time $\bar{T} = \max\{T_2, T_3\}$ the estimation of σ becomes exact and being made exclusively by the MIMO RED ($\alpha(\tilde{\nu}_{rl}) = 0$), which implies that $\varepsilon_g(t) = 0, \forall t \geq \bar{T}$.

In this case, the overall error system can be described by

$$\dot{x_e} = A_c x_e + B_c K_p[u - u^*] \,, \quad e_L = \bar{L} H_0 x_e \,, \tag{B.24}$$

since this system has uniform relative degree one we can apply the result obtained in [31]. Thus, it is possible to conclude that $z(t), e(t) \to 0$. ∎

Appendix C

eBMRAC proofs

C.1 Proof of Theorem 5

Property (i) is derived by considering the Lyapunov candidate: $2V_\theta = \theta^T\theta$. The derivative is:

$$\dot{V} = (\sigma_{eq} - \sigma)\,\|\theta\|^2 = (\sigma_{eq} - \sigma)V/2 \tag{C.1}$$

from (3.10) it follows that $(\sigma_{eq} - \sigma) \leq 0$ for $\|\theta\| \geq M_\theta$, such that $\|\theta(t)\| \leq M_\theta$ is positively invariant and therefore $\tilde{\theta}^T\tilde{\theta}$ is uniformly bounded.

Property (ii) is obtained using the following Lyapunov candidate:

$$V = x_\varepsilon^T P x_\varepsilon + \frac{1}{\gamma}\tilde{\theta}^T\tilde{\theta}$$

The time derivative is

$$\dot{V} = -x_\varepsilon^T Q x_\varepsilon - \frac{2\sigma}{\gamma}\tilde{\theta}^T\theta \tag{C.2}$$

Since $\|\theta\|$ is uniformly bounded

$$V \leq x_\varepsilon^T P x_\varepsilon + \mathcal{O}(\gamma^{-1}) \tag{C.3}$$

From which is possible to establish that

$$\dot{V} \leq -\lambda_1 \left[V - \mathcal{O}(\gamma^{-1})\right] \tag{C.4}$$

where $\lambda_1 = \lambda_{min}(Q)/\lambda_{max}(P)$, with $\lambda_{min}(Q)$ and $\lambda_{max}(P)$ being the minimum and maximum eigenvalues of Q and P, respectively. The proof of (ii) is completed using a comparison lemma.

Finally, property (iii) is established noting that without the projection, we would have $\dot{V} = -x_\epsilon^T Q x_\epsilon$. Consequently, $\dot{V} \leq -x_\varepsilon^T Q x_\varepsilon \leq 0$.

Since the term $-\frac{2\sigma}{\gamma}\tilde{\theta}^T\theta$ is non-positive due to the projection law it can only make

$\dot{V}$ more negative, such that it does not affect the obtained result. Furthermore, the $\mathcal{L}_2$ property is not affected by projection, since $\left|\dot{\theta}\right|^2 \leq \kappa \left|\gamma e_o \omega\right|^2$, where $\kappa > 0$ is a constant. Further details can be seen in [2] (pp. 205). Since $\varepsilon \to 0$. Thus,

$$\varepsilon = k^* W_m(s)[u_0 - u^*], \tag{C.5}$$

it is possible to establish that $u_0 \to u^*$. Consequently, referring to the tracking error:

$$e = k^* W_m(s) \left[\frac{u^*}{\tau s + 1} - u^*\right] \tag{C.6}$$

Which is equivalent to

$$e = k^* W_m(s) \left[\frac{-\tau s}{\tau s + 1}\right] u^* \tag{C.7}$$

Following similar steps presented in the proof of Theorem 2 in [71], it can be shown that ω is bounded and hence u^* is also bounded. Since $W_m(s)$ is minimum-phase, it follows that[1]

$$\left\| k^* W_m \frac{-\tau s}{\tau s + 1} \right\|_1 \leq \tau K_1 \tag{C.8}$$

thus $\|e\| \leq \tau K + c_2 e^{-\lambda_2 t}$ for some positive constants c_2 and λ_2.

C.2 Proof of Theorem 6

Similarly to [71], we consider $k^{nom} = k^* = 1$ for simplicity and write plant and disturbance as

$$y = G_p(s)u + W_d(s)d \tag{C.9}$$

where W_d is the transfer function from d to e with $u = \bar{u}^*$, where $\bar{u}^*$ is the control that achieves perfect matching with $d = 0$. It is then possible to write the output error as

$$e = y - y_m \tag{C.10}$$

$$= W_m(s)[-u_0 - u^*] + W_m(s)W_d(s)d + W_m(s)[u_0 - u] \tag{C.11}$$

The prediction error is then

$$\varepsilon = e - \hat{e} \tag{C.12}$$

$$= W_m(s)\left[u_0 - u^*\right] + W_m(s)W_d(s)d \tag{C.13}$$

[1]The $\mathcal{L}_1$ norm of the operator $h(t)$ corresponds to $\|h\|_1 = \int_0^\infty |x(\tau)| d\tau$

If the smooth saturation is used, the modified control signal is

$$u_0 = \theta^T \omega - \bar{d}\frac{\varepsilon}{\|\varepsilon\| + \delta}. \tag{C.14}$$

We rewrite as the prediction error as

$$\varepsilon = W_m(s)\left[\theta^T \omega - \bar{u}^*\right] + W_m(s)W_d(s)d - W_m(s)\bar{d}\frac{\varepsilon}{\|\varepsilon\| + \delta} \tag{C.15}$$

Using the following Lyapunov candidate

$$V = x_\varepsilon^T P x_\varepsilon + \frac{1}{\gamma}\tilde{\theta}^T \tilde{\theta} \tag{C.16}$$

Calculating its derivative, we have

$$V = 2\left(A_c\ x_\varepsilon + b_c\ [u_0 - u^*]\right)^T P x_\varepsilon + \frac{2}{\gamma}\left(-\sigma\theta - \gamma e\omega\right)^T \tilde{\theta} \tag{C.17}$$

which leads to

$$\dot{V} \leq -x_\varepsilon^T Q x_\varepsilon + 2x_\varepsilon^T P b_c\,[u_0 - u^*] - \frac{2\sigma}{\gamma}\theta^T\tilde{\theta} - 2\varepsilon\tilde{\theta}^T\omega + 2x_\varepsilon^T P b_c ||W_d|| d \tag{C.18}$$

If it is possible to cancel $\varepsilon\tilde{\theta}^T\omega$ with $x_\varepsilon^T P b_c\,[u_0 - u^*]$

$$\dot{V} \leq -x_\varepsilon^T Q x_\varepsilon - \frac{2\sigma}{\gamma}\theta^T\tilde{\theta} + 2x_\varepsilon^T P b_c ||W_d|| d + 2x_\varepsilon^T P b_c \bar{d}\frac{\varepsilon}{\|\varepsilon\| + \delta} \tag{C.19}$$

or, since the model is chosen SPR

$$\dot{V} \leq -x_\varepsilon Q x_\varepsilon - \frac{2\sigma}{\gamma}\theta^T\tilde{\theta} + \varepsilon^T ||W_d|| d + \bar{d}\frac{\varepsilon^T\varepsilon}{\|\varepsilon\| + \delta} \tag{C.20}$$

The rest of the proof follows closely to [73].

Appendix D

Publications

Until the present moment, this work is related to the following publications

Regarding the Direct MRAC:

- Transactions on Automatic Control regarding the Direct MRAC [56]
- 2013 American Control Conference [78]
- XIX Congresso Brasileiro de Automática, presented in 2012 [79]
- XX Congresso Brasileiro de Automática, presented in 2014 [56]

Regarding the GRED-BMRAC

- 13th Workshop on Variable Structure Systems [80]
- XX Congresso Brasileiro de Automática [81]

Regarding the eBMRAC

- 19th IFAC World Congress [82]
- XX Congresso Brasileiro de Automática [83]

Related topics

- XI Simpósio Brasileiro de Automação Inteligente [84]
- XX Congresso Brasileiro de Automática [85]